



UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics

Invitational B • 2009

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{49} = 7$$

**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $2(3 \times 4! \div (5 - 6) + 7^2 - 8)$

- (A) -132 (B) -62 (C) 9.6 (D) 63.8 (E) 66

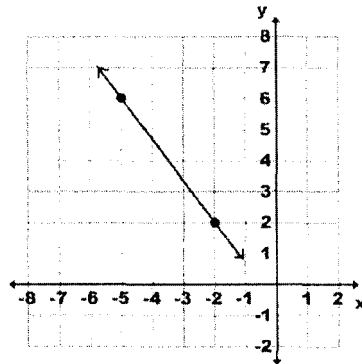
2. What is 25% of $\frac{3}{4}$ of 50 plus 75% of $\frac{5}{8}$ of 40?

- (A) 62.5 (B) 46.875 (C) 28.125 (D) 15.625 (E) 175.78125

3. The Cheep Choppe is having a February Sale. The regular price of their special coats is \$89.95. They are on sale for 30% off of the regular price. A newspaper coupon offers 10% off of the sale price. What would the selling price be if the customer brings in the coupon?

- (A) \$53.97 (B) \$56.67 (C) \$49.95 (D) \$71.98 (E) \$69.95

4. Find an equation of the line shown.



- (A) $3x + 4y = 2$ (B) $4x - 3y = 2$ (C) $3x + 3y = -2$ (D) $4x + 3y = -2$ (E) $4x + 4y = 2$

5. $(p - q) \times r = pr - qr$ is an example of which property of equality?

- (A) distributive (B) identity (C) inverse (D) subtraction (E) associative

6. Deputy Dawg is building two adjacent rectangular pens to hold his puppies. Each pen has a length 3 times longer than its width and the pens share a common side (width). He has 65 feet of fencing. What will the area of each pen be?

- (A) $64\frac{2}{3}$ sq. ft. (B) $63\frac{3}{8}$ sq. ft. (C) $60\frac{3}{4}$ sq. ft. (D) $58\frac{2}{3}$ sq. ft. (E) $56\frac{1}{3}$ sq. ft.

7. If a quadrilateral is inscribed in a circle, then its opposite angles are _____.

- (A) obtuse (B) congruent (C) supplementary (D) complementary (E) acute

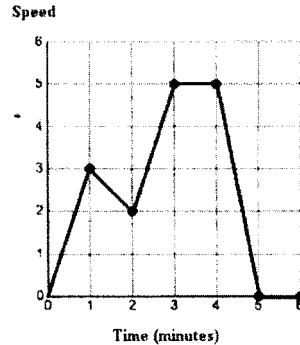
8. The coordinates of the vertices of $\triangle ABC$ are $(-2, 0)$, $(1, 4)$ and $(4, 0)$. The coordinates of the incenter is:

- (A) $(\frac{13}{16}, 1\frac{1}{4})$ (B) $(1, 1\frac{3}{16})$ (C) $(1\frac{3}{16}, 1\frac{1}{4})$ (D) $(1, 1\frac{1}{2})$ (E) $(1\frac{1}{4}, 1)$

9. Shirley Knott is filling up her circular wading pool. The diameter of the pool is 6 feet and the height of the pool is 1 foot. What is the maximum number of whole gallons of water can she use and not cause the pool to overflow?

- (A) 231 (B) 230 (C) 212 (D) 211 (E) 200

10. Betty Wheel rides her bicycle up and down the hilly streets from her house to school. The graph best depicts her 6 minute ride. (speed is not truly linear in this case). During the time interval of 2 minutes to 3 minutes Betty is _____.



- (A) going uphill (B) getting tired (C) going downhill
 (D) decreasing speed (E) increasing speed

11. Willie Dublett deposits \$500 in a bank account with an interest rate of 2.5% compounded monthly. How many months will it take for his balance to reach \$750?

- (A) 195 (B) 183 (C) 250 (D) 192 (E) 236

12. The 8th Fibonacci number is 13. The 10th Fibonacci number is 34. Find the 9th Lucas number.

- (A) 21 (B) 23.5 (C) 29 (D) 34 (E) 47

13. The vertex angle of an obtuse isosceles triangle has a measure of 100° and the length of one the sides adjacent to the vertex angle is 4 cm. Find the area of the triangle. (nearest tenth)

- (A) 13.9 cm^2 (B) 12.2 cm^2 (C) 10.3 cm^2 (D) 7.9 cm^2 (E) 6.1 cm^2

14. Determine the range of $f(x) = 2 + 3\cos(4x - 5)$.

- (A) $[-1, 5]$ (B) $[-2, 4]$ (C) $[-3, 3]$ (D) $[-1, 3]$ (E) $[-5, 3]$

15. A ramp is 18 ft. long and the angle of elevation of the ramp from the ground to the platform is $15^\circ 10' 5''$. Find the height of the platform. (nearest approximation)

- (A) $4' 7.9''$ (B) $4' 8.52''$ (C) $4' 8.56''$ (D) $4' 8.65''$ (E) $4' 8.72''$

16. Find L_9 if $L_0 = 2$, $L_1 = 1$, and $L_n = L_{(n-1)} + L_{(n-2)}$, where $n \geq 2$.

- (A) 29 (B) 47 (C) 76 (D) 34 (E) 55

17. Find $f(2)$ when $f(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$ (nearest thousandth)

- (A) $-.441$ (B) $-.408$ (C) $-.416$ (D) $-.466$ (E) $-.422$

18. If 5 men working 5 hours a day for 5 days can dig a tunnel 5 km in length, then how long of a tunnel can 10 men working 10 hours a day for 10 days dig?

- (A) 10 km (B) 15 km (C) 25 km (D) 40 km (E) 50 km

19. Let f be a function such that it is differentiable on (a,b) and continuous on $[a,b]$, and $f(a) = f(b) = 0$. Then there is a number c in (a,b) for which $f'(c) = 0$. This theorem is known as:

- (A) Sandwich Theorem (B) Least Limit Theorem (C) Rolle's Theorem
(D) Max–Min Theorem (E) L'Hopital's Rule

20. The point $P(2, 1)$ is rotated clockwise about the origin to point $(-1, -2)$. The angle of rotation, to the nearest degree, is:

- (A) 127° (B) 143° (C) 153° (D) 190° (E) 217°

21. The function $f(x) = \frac{2}{x-1} + 18x$ is increasing at which of the following values of x ?

- (A) $1\frac{2}{3}$ (B) $1\frac{1}{4}$ (C) 1 (D) $\frac{1}{12}$ (E) $\frac{1}{2}$

22. Roland Tuwin is playing a special dice game. He rolls two dice. If he rolls a double (1-1, 2-2, 3-3, etc.) he gets 20 points. If he does not roll a double and the sum of the dice is a prime number he gets 10 points. If he does not roll a double and the number is not a prime he loses 5 points. What is the mathematical expectation on any one roll?

- (A) 0 points (B) 5 points (C) 10 points (D) 15 points (E) 20 points

23. Betty Cheetz flips a fair coin and rolls a fair six-sided die. What are the odds that she will get a head and a prime number?

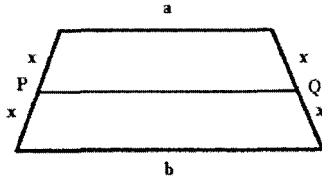
- (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{9}$ (D) $\frac{1}{6}$ (E) $\frac{1}{2}$

24. The universal set $U = \{2, 3, 5, 7, 11, 13, 15, 17, 19\}$. Subset $L = \{5, 7, 15, 17\}$, subset $M = \{3, 13\}$. How many elements are in the complement set of $L \cup M$?

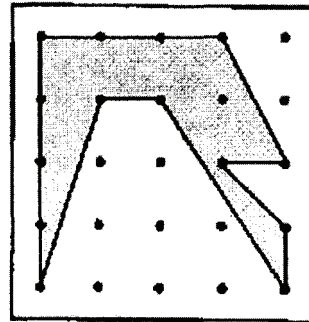
- (A) 3 (B) 4 (C) 6 (D) 7 (E) 9

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- (A) 3 (B) 4 (C) 6 (D) 7 (E) 9

25. Given the trapezoid shown with bases a and b , the length of segment PQ is the _____ mean of a and b .



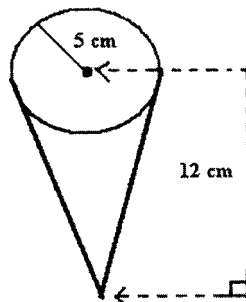
- (A) arithmetic (B) geometric (C) harmonic (D) half (E) quadratic
26. Which of the following mathematicians was most remembered as the inventor of logarithms?
- (A) Alicia Stott (B) Freda Porter (C) John Napier (D) Rene Descartes (E) Sophie Germain
27. If $x + y = -6$ and $xy = 9$ then $x^3 + y^3 = ?$
- (A) 567 (B) 513 (C) 378 (D) -27 (E) -54
28. Adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown.



- (A) 6.5 cm^2 (B) 7 cm^2 (C) 7.75 cm^2 (D) 8.5 cm^2 (E) 9 cm^2
29. The center of the circle, $x^2 + y^2 - 4x - 6y + 9 = 0$, is _____ units from the origin. (nearest tenth)
- (A) 2.5 (B) 3.0 (C) 3.6 (D) 5.0 (E) 7.2
30. A Ferris wheel has a radius of 7 meters and turns at 6 revolutions per minute. The bottom of the Ferris wheel is 1 meter above the ground. The height h of a passenger above the ground varies sinusoidally with time t . Which of the following equations best describes the relationship between h and t ?
- (A) $h = 6 + 7\cos(\frac{\pi}{6} t)$ (B) $h = 8 + 7\sin(\frac{\pi}{5} t)$ (C) $h = 7 - 6\cos(\frac{\pi}{5} t)$
- (D) $h = 7 - 8\sin(\frac{\pi}{3} t)$ (E) $h = 8 - 7\cos(\frac{\pi}{5} t)$
31. Find the area (in square units) of the region bounded by $y = -x^2$ and $y = -4$.
- (A) 16 (B) $11\frac{1}{3}$ (C) $10\frac{2}{3}$ (D) 8 (E) $5\frac{1}{3}$

32. Let $L = \{2, 1, 3, 4, 7, 11\}$. Two elements of set L are selected at random without replacement. What is the probability that the median of the two numbers selected is a whole number ?
- (A) $46\frac{2}{3}\%$ (B) $53\frac{1}{3}\%$ (C) 40% (D) $26\frac{2}{3}\%$ (E) $23\frac{1}{3}\%$
33. Lesleys Kwik runs the 400 meter dash at the local track meet. She runs the first 100 meters in 15 seconds, the second 100 meters in 16 seconds, the third 100 meters in 17.2 seconds and the last 100 meters in 18.5 seconds. What was her average speed? (nearest thousandth)
- (A) 5.997 m/sec (B) 6.016 m/sec (C) 6.034 m/sec (D) 6.052 m/sec (E) 6.228 m/sec
34. How many ordered pairs (x, y) are solutions to the equation $5x + 3y < 40$, where x, y are integers and $0 < y < x < 9$?
- (A) none (B) 6 (C) 10 (D) 14 (E) 21
35. Find the smallest integer k so that $4x^2 - 3x + k = 0$ has two imaginary roots.
- (A) -5 (B) -2 (C) -1 (D) 1 (E) 0
36. Let $f(x) = 2x + 1$ and $g(x) = 4 - 3x$, then $f^{-1}[g^{-1}(-1)]$ equals:
- (A) -7 (B) $-1\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $2\frac{1}{3}$ (E) 13
37. If $p + q = 12$ and $p \times q = 22$ then $(p - q)^2 = ?$
- (A) 34 (B) 56 (C) 100 (D) 340 (E) 436
38. Noah Kanwen won 40 of 75 games. How many of the next 25 games can Noah lose in order to have won 60% overall?
- (A) 23 (B) 20 (C) 15 (D) 13 (E) 5
39. Three students in Miss Woik's class were absent the day of the exam. The average of the other 12 students was 84. What would the three absent students have to average on their make-up exam in order to bring the entire class average to 86?
- (A) 85 (B) 88 (C) 91 (D) 94 (E) 99
40. Two chords, WY and XZ intersect in the interior of a circle at point P such that $m\angle WPX = 70^\circ$ and $m\widehat{WX} = 120^\circ$. If points X and Y are not on \widehat{WZ} then $m\widehat{YZ}$ is:
- (A) 20° (B) 25° (C) 35° (D) 60° (E) 95°

41. Find the lateral area, nearest square cm, of the cone.



- (A) 188 cm^2 (B) 204 cm^2 (C) 245 cm^2 (D) 283 cm^2 (E) 377 cm^2

42. Which of the following is true about the relation $f(x) = x^2 + 2x + 2$?

- (A) odd function (B) even function (C) neither even nor odd function
 (D) not a function (E) none of these are true

43. Find the determinant:
$$\begin{bmatrix} -1 & 2 & 3 \\ 1 & -2 & 3 \\ 1 & 2 & -3 \end{bmatrix}$$

- (A) -6 (B) -5 (C) 6 (D) 18 (E) 24

44. How many leaves are in the "rose" curve graph of the polar equation $r = 3 - 4\sin(2\theta + 5)$?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

45. Which of the following is a reference angle for 1645° ?

- (A) 45° (B) 25° (C) 22.5° (D) 18° (E) 4.5°

46. Simplify: $\frac{(n-1)!(n+2)!}{(n+1)!(n-2)!}$

- (A) $(n-1)^2$ (B) $\frac{n+1}{n-1}$ (C) $\frac{n+2}{n-2}$ (D) $n^2 - 2$ (E) $n^2 + n - 2$

47. How many different ways can you select 5 bills from a cash box containing \$1, \$2, \$5, \$10, \$20, \$50, and \$100 bills?

- (A) 330 (B) 462 (C) 792 (D) 1,584 (E) 1,980

48. Let $f(x) = \frac{4x+5}{3x}$. Find $f'(2)$.

- (A) $-1\frac{2}{3}$ (B) $-\frac{5}{12}$ (C) $-\frac{1}{12}$ (D) $1\frac{1}{3}$ (E) $2\frac{1}{6}$

49. Find an equation of the tangent line to the curve $y = \sqrt{9 - 4x}$ at the point $(-4, 5)$.

(A) $5x - 2y = -30$

(B) $2x - 5y = 33$

(C) $-2x + 5y = -10$

(D) $2x - 5y = -10$

(E) $2x + 5y = 17$

50. A bag contains yellow golf balls and orange golf balls. The probability of selecting a yellow ball is $\frac{2}{5}$. If 20 yellow balls are added to the bag, the probability of selecting a yellow ball becomes $\frac{4}{7}$. How many orange balls are in the bag?

(A) 10

(B) 14

(C) 30

(D) 35

(E) 42

51. Simplify: $\left((a^2b)^{-3} \times (ab^2) \div (a^2b^{-3}) \times (ab) \right)^{-1}$, where $a, b > 0$.

(A) $a^6 b^{-3}$

(B) $a^{-5} b^3$

(C) $a^8 b^3$

(D) $a^{-3} b^2$

(E) $a^{-2} b^3$

52. In 3 years Sid Upp will be twice as old as his son, Stan Upp. Five years ago Stan's age was $\frac{1}{3}$ of his father's age at that time. What is the sum of their ages now?

(A) 48

(B) 42

(C) 37

(D) 32

(E) 29

53. Point $P(2, -3)$ is reflected across the origin to point Q . Then point Q is translated horizontally 3 units to the right to point R . Point R is reflected across the origin to point S . The coordinates of point S is (x, y) . Find $x + y$.

(A) 4

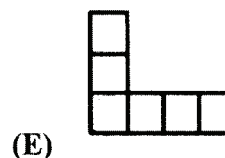
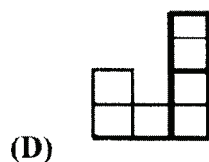
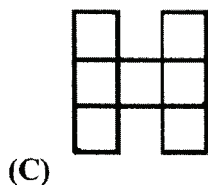
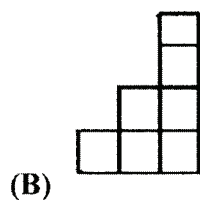
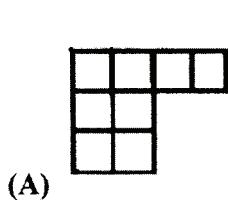
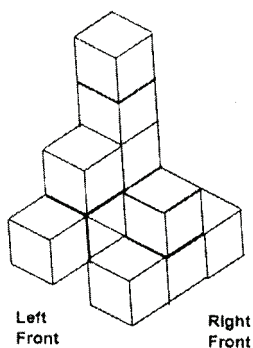
(B) 1

(C) 0

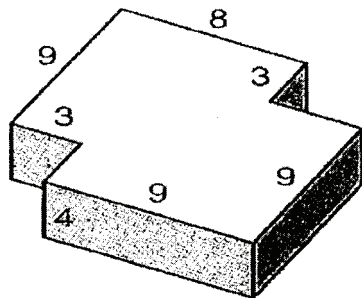
(D) -1

(E) -4

54. Which of the following would best represent a two dimensional perspective of the front right side view of this figure shown?



55. If you start at $(-1.5, 0)$ on the x -axis and travel horizontally 12 radians to the right, how many times will you cross the graph of $y = \sin(3x)$?
- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15
56. If $9^{(x+2y)} = 81$ and $9^{(2x-y)} = \frac{1}{9}$, then $3^{xy} = ?$
- (A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{9}$ (E) 1
57. Tye Guhr drops a golf ball from a height of 10 feet. It bounces back to a height of 60% of the distance it fell. How far has it traveled when it hits the ground the fourth time? (nearest inch)
- (A) 43' 5" (B) 33' 6" (C) 31' 4" (D) 25' 7" (E) 23' 6"
58. The point of inflection on the graph of $f(x) = 2x^3 - 6x^2 + 6x - 6$ is (a, b) . Find b .
- (A) 2 (B) 1 (C) 0 (D) -4 (E) -6
59. The Buddy System motorcycle testing company is testing a motorcycle with a side car. They hire 4 cyclists to do the testing in pairs. How many arrangements of driver and rider are possible?
- (A) 4 (B) 6 (C) 8 (D) 12 (E) 24
60. Find the lateral surface area of this prism. All angles are right angles.



- (A) 188 units² (B) 315 units² (C) 438 units³ (D) 246 units² (E) 192 units²

**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation B • 2009
Answer Key**

- | | | |
|-------|-------|-------|
| 1. B | 21. A | 41. B |
| 2. C | 22. B | 42. C |
| 3. B | 23. A | 43. E |
| 4. D | 24. A | 44. D |
| 5. A | 25. A | 45. B |
| 6. E | 26. C | 46. E |
| 7. C | 27. E | 47. B |
| 8. D | 28. B | 48. B |
| 9. D | 29. C | 49. E |
| 10. E | 30. E | 50. C |
| 11. A | 31. C | 51. A |
| 12. E | 32. A | 52. B |
| 13. D | 33. A | 53. E |
| 14. A | 34. D | 54. B |
| 15. B | 35. D | 55. A |
| 16. C | 36. C | 56. E |
| 17. C | 37. B | 57. B |
| 18. D | 38. E | 58. D |
| 19. C | 39. D | 59. D |
| 20. B | 40. A | 60. E |