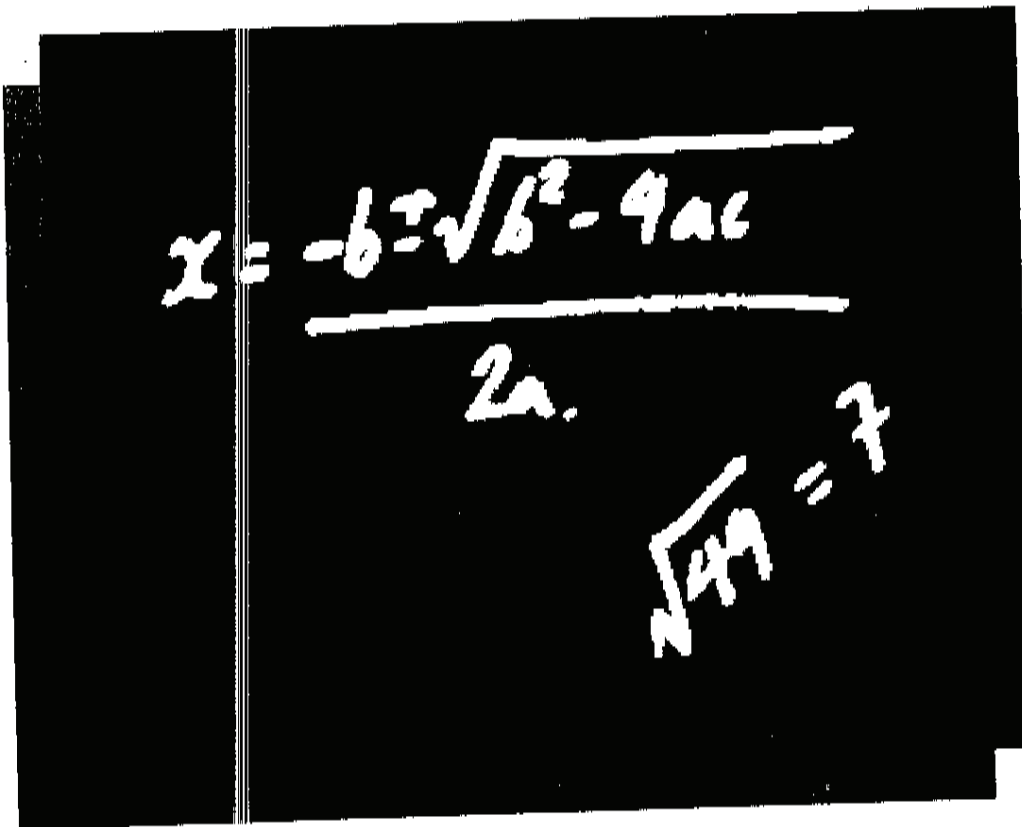




UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics

District 1 • 2009



**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $2 \div \left[\left(\frac{2}{3}\right)^2 - (3)^{-2} \right] \times 3!$

- (A) 36 (B) $26\frac{1}{3}$ (C) $21\frac{3}{5}$ (D) 4 (E) 1

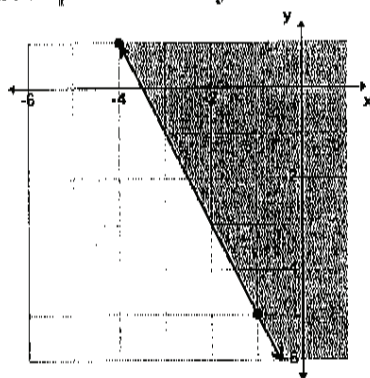
2. $3A + 0 = 3A$ and $3B \times 1 = 3B$ are examples of the _____ properties of equality.

- (A) associative (B) commutative (C) distributive (D) identity (E) inverse

3. In three years Odie Butgoody's age will be six times what Yung Su's age was last year. Two years ago Odie's age was five times what Yung Su's age now. What is the sum of their ages now?

- (A) 68 (B) 65 (C) 60 (D) 57 (E) 55

4. The shaded area is best represented by which of the following inequalities?



- (A) $4x + 2y \geq -3$ (B) $2x - y \leq 7$ (C) $2x + y \geq -6$
 (D) $4x + 2y \leq 1$ (E) $2x + y \geq -7$

5. Shirley Knott is filling up her circular wading pool. The radius of the pool is 4 feet. How deep will the water be if she puts 250 gallons of water in the pool? (nearest inch)

- (A) 5" (B) 6" (C) 7" (D) 8" (E) 9"

6. A segment extending from the vertex angle of an isosceles triangle to the midpoint of the opposite side is called a(n):

- (A) altitude (B) bisector (C) centroid (D) directrix (E) mean

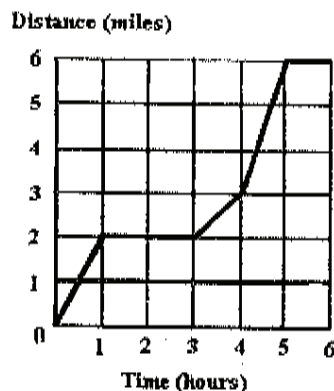
7. The center of the circle $x^2 + y^2 - 4x + 10y + 12 = 0$ is:

- (A) (4, -10) (B) (-2, 6) (C) (2, -5) (D) (6, -5) (E) (6, -4)

8. Which of the following is one of the few American Indian women to earn a Ph.D. in applied mathematics and computational sciences? She is a member of the Lumbee tribe.

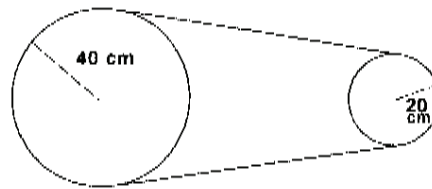
- (A) Emmy Noether (B) Freda Porter (C) Hypatia
 (D) Karen E. Smith (E) Grace Alele Williams

9. The graph best depicts Ima Hiker's 6 hour backpack trip. During which of the following time intervals was Ima walking the fastest?

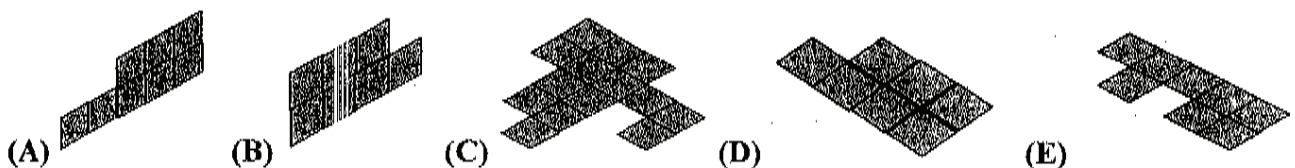
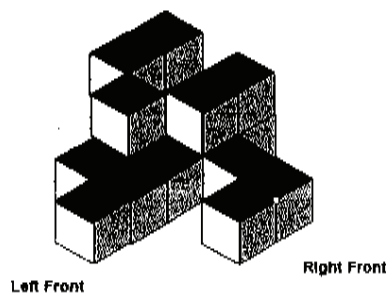


- (A) 0 - 1 (B) 1 - 3 (C) 3 - 4 (D) 4 - 5 (E) 5 - 6
10. How many 2-member subsets and 4-member subsets can be made from the set $\{\clubsuit, \triangle, \heartsuit, \oplus, \diamond, \square\}$?
- (A) 35 (B) 30 (C) 21 (D) 20 (E) 15
11. Petunia's garden shop has three varieties of tomato plants, Big Boy, Cherry, and Porter. They have 5 Big Boy plants, 4 Cherry plants and 7 Porter plants mixed together without labels. The young plants all look alike. Petunia randomly selects three plants. What is the probability that the three she selects are the same variety?
- (A) 7.97 % (B) 2.58 % (C) 8.75 % (D) 3.42 % (E) 5.62 %
12. Suppose that $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = ?$
- (A) $f'(a) - g'(a)$ (B) $\frac{f'(a)}{g'(a)}$ (C) 0 (D) $[f'(a)][g'(a)]$ (E) $\frac{f(a)}{g(a)}$
13. Let $f(x) = \sin x \cos x$. Find $f'(\frac{\pi}{3})$.
- (A) $-\frac{1}{2}$ (B) $-\sqrt{3}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{3}{2}$
14. Find $x + y$ if $\begin{bmatrix} x & -2 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} 7 \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.
- (A) 5 (B) 4 (C) 3 (D) -4 (E) -7
15. Which of the following is true about the relation $f(x) = \frac{x^2 - 1}{x^4 + 1}$?
- (A) odd function (B) even function (C) neither even nor odd function
 (D) not a function (E) one-to-one function

16. A belt joins the two pulleys shown. The smaller pulley with the given radius revolves at the rate of 48 rpm. Find the linear velocity of the belt in centimeters per minute. (nearest tenth)



- (A) 1508.0 (B) 3055.8 (C) 4523.9 (D) 6031.9 (E) 6111.5
17. Point P has polar coordinates of $(3\sqrt{3}, \frac{5\pi}{3})$. What are the rectangular coordinates of point P?
- (A) $(1\frac{2}{3}, -4\frac{1}{2})$ (B) $(-4\frac{1}{2}, -\frac{\sqrt{3}}{2})$ (C) $(5, 3\sqrt{3})$ (D) $(\frac{\sqrt{3}}{3}, 5)$ (E) $(\frac{3\sqrt{3}}{2}, -\frac{9}{2})$
18. Last year the cost of a gallon of gas was \$3.75. This year a gallon of gas costs \$1.55. Phil Upp's car gets 20 miles per gallon of gas and he has \$15.00 to spend on gas. How much farther can Phil drive this year than he could last year? (nearest whole mile)
- (A) 174 miles (B) 114 miles (C) 87 miles (D) 80 miles (E) 44 miles
19. The set $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34 \dots\}$ is closed under which of the following operations :
- I. addition II. subtraction III. multiplication IV. division
- (A) all of these (B) I & III only (C) I, II, & III (D) II & IV only (E) none of these
20. The circles $(x - 2)^2 + (y + 5)^2 = 17$ and $(x - 4)^2 + (y + 2)^2 = 28$ intersect in two points. The slope of the line through the two points of intersection is:
- (A) $-\frac{1}{5}$ (B) $-\frac{2}{3}$ (C) $-3\frac{1}{3}$ (D) 1 (E) $1\frac{1}{2}$
21. Which of the following would best represent a two dimensional perspective of the right front view of this figure shown?

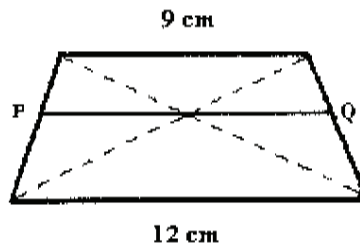


22. If $\cos x + \sin x = .5$ then the approximate value of $\tan x$ is:
- (A) -1.134 (B) -0.268 (C) -0.451 (D) -0.577 (E) -2.679
23. Dr. J drops a basketball ball from a height of 9 feet. It bounces back to a height of 40% of the distance it fell. How far has it traveled when it hits the ground the fourth time? (nearest inch)
- (A) 14' 2" (B) 14' 7" (C) 19' 8" (D) 20' 3" (E) 20' 8"
24. Find the equation of the line tangent to the curve $y = x^3 - 2x^2 + 1$ at $x = -1$.
- (A) $y = 1 - 2x$ (B) $y = -2$ (C) $y = 7x + 5$ (D) $y = 2x - 1$ (E) $y = 7x + 1$
25. Find the sum of all numbers between 0 and 250 that are multiples of 8.
- (A) 4000 (B) 3948 (C) 3,906 (D) 3,968 (E) 4,096
26. The integers greater than 1 are arranged in five columns as shown. If this pattern continues which column would contain the number 725?

A	B	C	D	E
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	

- (A) A (B) B (C) C (D) D (E) E
27. Minnie Mumm has a sheet of metal that is 3 feet by 5 feet. She is going to cut out the same size square from each of the four corners, then fold up the sides, solder them, and make a rectangular cistern that will hold the greatest amount of water. What percent of the sheet of metal will she cut out and not use? (nearest tenth of a percent)
- (A) 7.4 % (B) 8.4 % (C) 8.9 % (D) 9.8 % (E) 10.5 %
28. The function $f(x) = \frac{3}{2x-1} + 24x$ is decreasing at which of the following values of x ?
- (A) $\frac{5}{6}$ (B) $\frac{5}{7}$ (C) $\frac{7}{9}$ (D) $\frac{4}{5}$ (E) $\frac{9}{11}$
29. Simplify $[\sin \theta + \cos \theta \times \cot \theta] \times \tan \theta$.
- (A) $\sin \theta - \cos \theta$ (B) $\sec \theta$ (C) $\sin \theta \cos \theta$ (D) $\csc \theta$ (E) $\sin \theta + \cos \theta$
30. Determine the range of $f(x) = 5 - 4\cos(3x + 2)$.
- (A) [1, 9] (B) [-2, 6] (C) [3, 7] (D) [-1, 3] (E) [-4, 1]

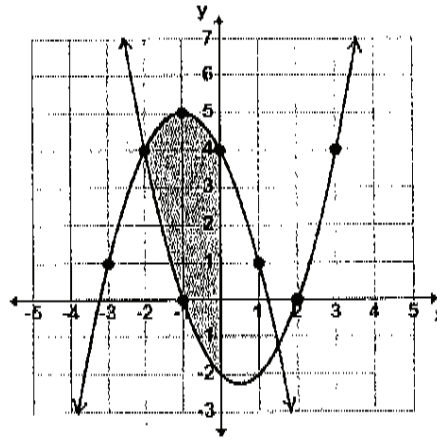
31. Find the length of segment PQ in the trapezoid shown. (nearest hundredth)



- (A) 10.00 cm (B) 10.29 cm (C) 10.39 cm (D) 10.50 cm (E) 10.61 cm
32. $\sum_{k=0}^2 (-1)^k x - (-1)^k = ?$
- (A) $3x + 1$ (B) $x - 3$ (C) $2x - 1$ (D) -1 (E) $x - 1$
33. The universal set $L = \{1, 3, 4, 7, 11, 18, 29, 47, 76\}$. Subset $E = \{4, 18, 76\}$, subset $P = \{3, 7, 11, 29, 47\}$. How many elements are in the complement set of $E \cup P$?
- (A) 9 (B) 8 (C) 5 (D) 3 (E) 1
34. Willie Cheet flips two fair coins and rolls a fair six-sided die. What are the odds that he will get a head, a tail, and a Fibonacci number?
- (A) $\frac{1}{9}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$
35. Simplify: $\frac{2x-4}{x^2-4} \times \frac{4x^2+4x+1}{2x-2} \div \frac{2x^2-3x-2}{x+2}$
- (A) $\frac{2x^2-x-1}{x-2}$ (B) $\frac{2x+1}{2x^2-3x-2}$ (C) $\frac{x-1}{x-2}$ (D) $\frac{2x+1}{x^2-3x+2}$ (E) $\frac{x-2}{2x+1}$
36. Which of the following is NOT a solution to $\frac{3|2x+4|}{5} < 1$?
- (A) $-2\frac{2}{3}$ (B) -2 (C) $-1\frac{1}{8}$ (D) $-2\frac{1}{6}$ (E) $-1\frac{1}{5}$
37. Hickory, Dickory, and Doc were looking at their circular clock. The time was precisely 4:44 am. They calculated the measure of the smaller angle formed by the minute hand and the hour hand of the clock to be:
- (A) 144° (B) 120° (C) 140° (D) 130° (E) 122°
38. A segment extending from the edge of an ellipse to the center of the ellipse and is perpendicular to the major axis is called the _____.
- (A) asymptote (B) directrix (C) eccentricity (D) focus (E) semiminor axis

39. The Fibonacci numbers (denoted by F) are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144
The Lucas numbers (denoted by L) are 2, 1, 3, 4, 7, Find L_{11} .
- (A) 89 (B) 123 (C) 144 (D) 199 (E) 322
40. $4_5 \times 33_4 - 222_3 = \underline{\hspace{2cm}}_2$
- (A) 1011010 (B) 10010 (C) 1111 (D) 101101 (E) 100010
41. Trudy Frudy wants to mix some fruit ade that is 92.5% fruit with a special solution that is only 90% fruit. How many grams of fruit ade will she need to mix with the special solution to create 500 grams of 91% fruit?
- (A) 200 g (B) 225 g (C) 250 g (D) 275 g (E) 300 g
42. Two chords, PS and QR intersect in the interior of a circle at point C such that $m\widehat{QS} = 40^\circ$ and $m\angle PCR = 60^\circ$. If points R and S are not on \widehat{PQ} then $m\widehat{PR}$ is:
- (A) 20° (B) 50° (C) 80° (D) 100° (E) 120°
43. Let $f(x) = 3x + 2$, $g(x) = 4 - 5x$, and $h(x) = x - 1$ then $h\left[f\left(g\left[1\right]\right)\right]$ equals:
- (A) -26 (B) -2 (C) -1 (D) 0 (E) 14
44. Fuzzy Hare raises rabbits. He has 3 brown rabbits, 5 white rabbits, and 8 multicolored rabbits. How many ways can he put rabbits in a cage that will hold 1 brown rabbit, 2 white rabbits, and 3 multicolored rabbits?
- (A) 8008 (B) 3360 (C) 840 (D) 7200 (E) 1680
45. Which of the following is a focus for $3x^2 - y^2 - 6x + 2y = 10$.
- (A) (3, 1) (B) (-4, -1) (C) (5, 1) (D) (-4, 2) (E) (-1, 1)
46. Simplify: $\frac{(n-2)!(n+1)!}{(n-1)!n!}$
- (A) $n^2 + n$ (B) $\frac{n+1}{n-1}$ (C) $\frac{n-2}{n+2}$ (D) $\frac{n+1}{n-2}$ (E) $\frac{n^2+n}{n-1}$
47. Hope Imakett is staking out a triangular garden in a big field. She puts a stake in the ground where she is standing. Then she walks 25 feet on a bearing of 75° where she puts a second stake. She walks another 25 feet on a bearing of 150° where she puts the third stake. What is the perimeter of Hope's garden? (nearest foot)
- (A) 69 feet (B) 80 feet (C) 85 feet (D) 90 feet (E) 96 feet

48. Find the area of the shaded region in square units.



- (A) $7\frac{5}{6}$ (B) $8\frac{1}{3}$ (C) $8\frac{1}{2}$ (D) $8\frac{2}{3}$ (E) $9\frac{1}{6}$

49. Which of the following is a reference angle for -2400° ?

- (A) 15° (B) 30° (C) 45° (D) 60° (E) 75°

50. Harry Green can mow the #1 fairway in 1.5 hours. If Harry hires Sandy Trappe to help him they can mow it in 1 hour working together. How long would it take Sandy to mow it by herself?

- (A) 1 hour (B) 1.5 hours (C) 2 hours (D) 2.5 hours (E) 3 hours

51. The point $(-1, -2)$ is rotated 210° counterclockwise about the origin to point P. Which of the following points is the closest approximation to point P.

- (A) $(-0.13, 2.23)$ (B) $(1.87, -1.23)$ (C) $(-0.50, 0.87)$ (D) $(1.87, -2.23)$ (E) $(-0.13, 1.87)$

52. Snow White's 7 dwarfs are the only contestants in a beauty contest, in how many ways can She award 1st place, 2nd place and 3rd place trophies?

- (A) 336 (B) 180 (C) 216 (D) 343 (E) 210

53. Ye Olde Arte has a large supply of colored poster board. The colors are blue, green, beige, white, yellow, and red. How many different ways can the store package 5 poster boards?

- (A) 126 (B) 231 (C) 252 (D) 378 (E) 462

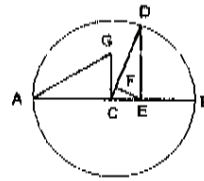
54. Let $f(x) = 2x - 5$ and $g(x) = f(1 - x)$. Find $g^{-1}(x)$.

- (A) $-(2x + 3)$ (B) $\frac{x-2}{3}$ (C) $3x - 2$ (D) $3 + 2x$ (E) $\frac{-x-3}{2}$

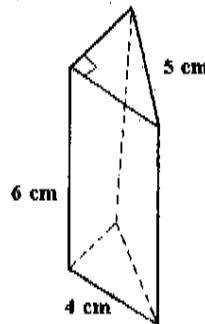
55. The coordinates of the vertices of $\triangle ABC$ are $(-3, -2)$, $(3, -1)$ and $(0, 3)$. The coordinates of the centroid is:

- (A) $(\frac{1}{3}, \frac{2}{3})$ (B) $(-\frac{1}{2}, -\frac{1}{3})$ (C) $(2, 0)$ (D) $(-1\frac{2}{3}, 0)$ (E) $(0, 0)$

56. Let AB be the diameter of the circle with center C with $CG \perp AB$, $DE \perp AB$, and $EF \perp DC$. If $AE = 10$, $BE = 6\frac{2}{5}$, and $CG = 1\frac{4}{5}$ then $DE = ?$



- (A) $3\frac{2}{5}$ (B) $4\frac{1}{2}$ (C) $4\frac{3}{5}$ (D) 8 (E) $8\frac{1}{5}$
57. The variable P varies jointly as the values of Q and R . Find the constant of proportionality if $P = 88$ when $Q = 4$ and $R = \frac{2}{5}$.
- (A) $35\frac{1}{5}$ (B) 55 (C) 64 (D) $140\frac{4}{5}$ (E) 220
58. Simplify: $(ab^2)^{-1} \times (a^2b^2)^{-3} \div (a^{-1}b^{-1}) \times a \div b$, where $a, b > 0$.
- (A) $\frac{1}{a^7b^6}$ (B) $\frac{1}{a^8b^{10}}$ (C) $\frac{1}{a^5b^8}$ (D) $\frac{1}{a^7b^{10}}$ (E) $\frac{1}{a^5b^6}$
59. Find the product of the arithmetic mean 3, 5, & 8, the geometric mean of 3, 5, & 8, and the harmonic mean of 3, 5, & 8. (nearest whole number)
- (A) 120 (B) 153 (C) 174 (D) 3657 (E) 3920
60. Find the lateral area, nearest square cm, of the right prism.



- (A) 66 cm^2 (B) 72 cm^2 (C) 78 cm^2 (D) 84 cm^2 (E) 99 cm^2

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 1 • 2009
Answer Key**

- | | | |
|-------|-------|-------|
| 1. A | 21. A | 41. A |
| 2. D | 22. C | 42. C |
| 3. A | 23. D | 43. B |
| 4. E | 24. C | 44. E |
| 5. D | 25. D | 45. C |
| 6. A | 26. E | 46. B |
| 7. C | 27. D | 47. D |
| 8. B | 28. B | 48. D |
| 9. D | 29. B | 49. D |
| 10. B | 30. A | 50. E |
| 11. C | 31. B | 51. A |
| 12. B | 32. E | 52. E |
| 13. A | 33. E | 53. C |
| 14. A | 34. E | 54. E |
| 15. B | 35. D | 55. E |
| 16. D | 36. C | 56. D |
| 17. E | 37. E | 57. B |
| 18. B | 38. E | 58. C |
| 19. E | 39. D | 59. A |
| 20. B | 40. E | 60. B |