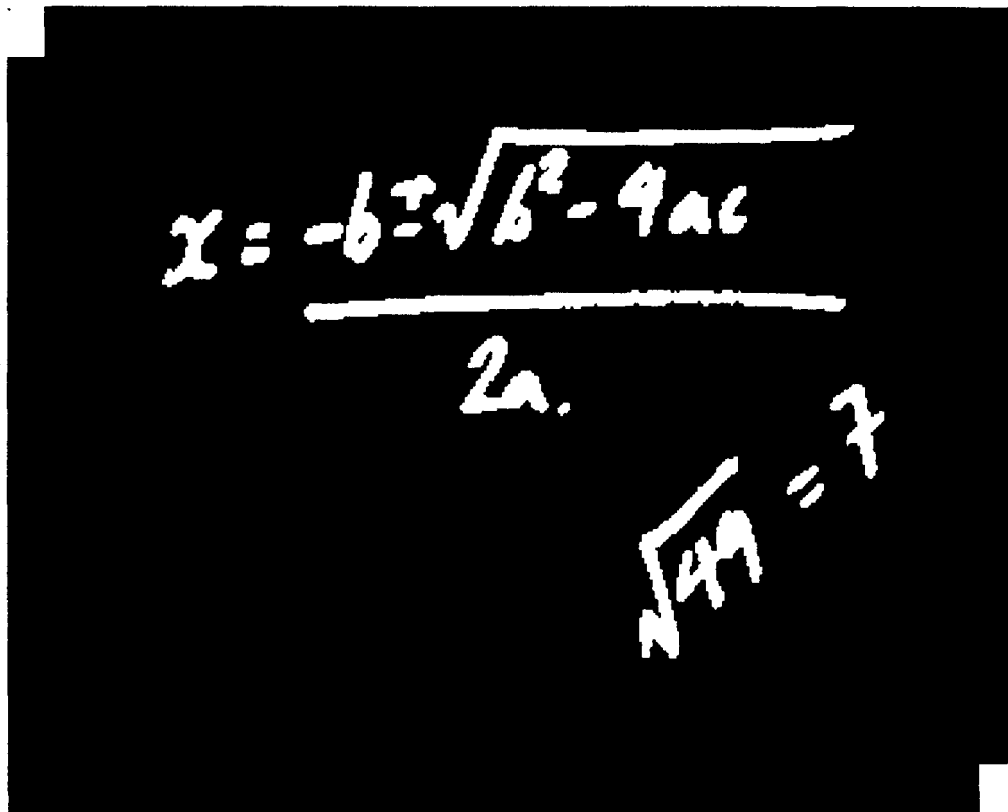




UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics

State • 2009

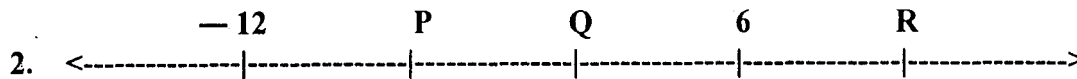


**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $0.875 + (\frac{3}{4} - 0.625 \times \frac{1}{2}) \div 0.375 + \frac{1}{4} \div 0.125$

- (A) $4\frac{1}{24}$ (B) 3.4375 (C) $4\frac{1}{2}$ (D) 4.375 (E) $3\frac{1}{6}$



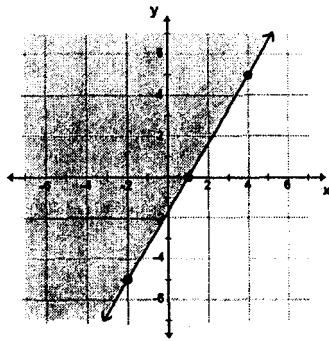
The distances between the hash marks (|) are equal. Find $\frac{R}{P-Q}$.

- (A) -6 (B) -2 (C) $\frac{1}{2}$ (D) 1 (E) 2

3. If $2x + 3 = 5$ and $5 = 6x - 1$ then $2x + 3 = 6x - 1$ is an example of the _____ property.

- (A) reflexive (B) commutative (C) symmetric (D) transitive (E) distributive

4. The shaded area is best represented by which of the following inequalities?



- (A) $y \leq \frac{5x-3}{5}$ (B) $y \geq \frac{3x-3}{5}$ (C) $y \geq \frac{5-5x}{3}$ (D) $y \leq \frac{3-5x}{5}$ (E) $y \geq \frac{5x-5}{3}$

5. If $\frac{4x}{7} + \frac{3}{5y} = \frac{x}{2}$, then xy equals _____.

- (A) -4.2 (B) -5.3 (C) -6.1 (D) -8.4 (E) -10.2

6. $\angle A$ and $\angle B$ are complementary angles. If $m\angle A = 2x - 3$ and $m\angle B = 4x + 5$, the measure of the supplement to the larger angle is:

- (A) $146\frac{2}{3}^\circ$ (B) $116\frac{1}{3}^\circ$ (C) $127\frac{1}{3}^\circ$ (D) $63\frac{2}{3}^\circ$ (E) $153\frac{1}{3}^\circ$

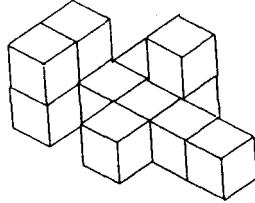
7. Sweetwater's water tank is in the shape of a sphere. The radius of the tank is 12 feet. How many gallons of water will the tank hold? (nearest gallon)

- (A) 7,238 gal (B) 81,219 gal (C) 13,536 gal (D) 162,437 gal (E) 54,146 gal

8. The point $(-4, 4)$ is reflected across the line $y = x$ to point P. Point P is rotated 90° clockwise about the origin to point Q. Point Q is translated 4 units horizontally to the right point R. Point R is reflected across the line $y = -x$ to point S. Where does point S lie?

- (A) x-axis (B) QI (C) QII (D) QIII (E) QIV

9. One-centimeter cubes are glued together to form the object in the figure shown. The two-dimensional perspective of the top view of this figure has a perimeter of:

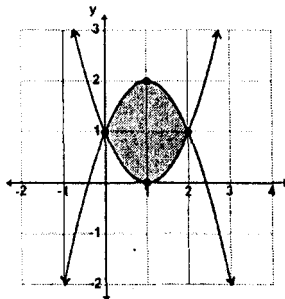


- (A) 27 cm (B) 23 cm (C) 21 cm (D) 18 cm (E) 16 cm
10. The ratio of the distance between the foci to the length of the major axis of an ellipse is called the _____.
- (A) asymptote (B) directrix (C) eccentricity (D) focal distance (E) semiminor axis
11. Let $f(x) = 3 - 4x$ and $g(x) = 2x + 1$. Find $f(g^{-1}(f(x + 1)))$.
- (A) $32x + 7$ (B) $-2x + 3$ (C) $8x - 1$ (D) $8x + 7$ (E) $-32x - 1$
12. Let $f(x) = \frac{1}{2} + \frac{3}{2} \sin(4x + \frac{4\pi}{3})$. Which of the following has a value of $\frac{2}{\pi}$?
- (A) amplitude (B) displacement (vertical) (C) frequency (D) phase shift (E) period
13. A Cell tower is 500 feet from a TV tower. The angle of elevation from the top of the Cell tower to the top of the TV tower is 30° . The angle of depression from the top of the Cell tower to the bottom of the TV tower is 40° . How tall is the TV tower? (nearest foot)
- (A) 885 ft (B) 847 ft (C) 789 ft (D) 708 ft (E) 689 ft
14. How many asymptotes does $f(x) = \frac{x^3 - 1}{x^2 - x - 2}$ have?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
15. Mr. White wants to buy 3 different pepper plants from Juanita's Hot Pepper Shop. She has 5 Jalapeno plants, 4 Hot Banana plants, and 3 Serrano plants to choose from, but she does not know which plant is which type. She randomly picks 3 plants to sell to Mr. White. What is the probability that the three she selects are all different?
- (A) $4\frac{6}{11}\%$ (B) $9\frac{1}{11}\%$ (C) $20\frac{5}{11}\%$ (D) $27\frac{3}{11}\%$ (E) $30\frac{9}{11}\%$
16. It takes 7 minutes for Phil DeTubb to fill his bathtub using the hot water faucet. It only takes 5 minutes using the cold water faucet. Phil turns on the hot water faucet. After 1 minute he turns on the cold water faucet and runs both faucets together. How long will it take to fill the bathtub? (nearest second)
- (A) 3 min 5 sec (B) 3 min 30 sec (C) 3 min 44 sec (D) 3 min 50 sec (E) 3 min 55 sec

17. Find $a + b + c$ given the arithmetic sequence: $a, -1.1, b, 3.3, c, \dots$

- (A) 0 (B) 1.1 (C) 3.3 (D) 6.6 (E) 9.9

18. Find the area of the shaded region in square units.



- (A) $2\frac{3}{4}$ (B) $2\frac{2}{3}$ (C) $2\frac{1}{2}$ (D) $2\frac{1}{3}$ (E) 2

19. Use the angle of rotation, θ (nearest degree), where $0^\circ < \theta < 90^\circ$, to transform the conic $x + 2xy - y = 0$ into an equation that is in standard position and does not contain an xy term. The transformed equation is:

- (A) $x^2 = y^2 + \sqrt{2}y$ (B) $x^2 = y^2$ (C) $x^2 = y^2 - \sqrt{2}y$
 (D) $y^2 = \sqrt{2}x - x^2$ (E) $y^2 = x^2 + \sqrt{2}x$

20. The curve of a polar equation $r = a + b \cos \theta$ is called a dimpled limaçon when which of the following is true:

- (A) $a = b$ (B) $a < b < 2a$ (C) $a < b$ (D) $2a \leq b$ (E) $a > b$

21. The Austin Bats has 3 coaches, 10 infielders and 6 outfielders. A committee consisting of 1 coach, 3 infielders, and 2 outfielders is selected. How many different committees could be selected?

- (A) 27,132 (B) 4,522 (C) 145 (D) 1,800 (E) 5,400

22. How many subsets containing an odd number of members can be made from the set $\{\Omega, \Phi, \Pi, \Psi, \Sigma, \Theta\}$?

- (A) 30 (B) 32 (C) 35 (D) 41 (E) 42

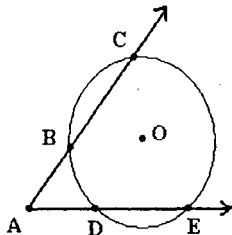
23. Coach Venn has 30 students on his math team. Sixteen students want to do math, 16 want to do calculator, and 11 want to do number sense. Five want to do both math and number sense. Three of those 5 want to do calculator as well. Eight students want to do calculator only and 5 want to do number sense only. How many students want to do math only?

- (A) 9 (B) 8 (C) 7 (D) 4 (E) 1

24. The sum of the real solutions of $5 - |4x + |3 - 2x|| = 1$ is:

- (A) $-3\frac{2}{3}$ (B) -3 (C) -2 (D) 1 (E) $1\frac{2}{3}$

25. Using the figure below, if $m\angle BAD = 50^\circ$ and $m\widehat{CE} = 150^\circ$ then $m\widehat{BD} = \underline{\hspace{2cm}}$.



- (A) 25° (B) 35° (C) 50° (D) 60° (E) 75°

26. The integers greater than 1 are arranged in five columns as shown. If this pattern continues which column would contain the number 2010?

A	B	C	D	E
	1	2	3	4
8	7	6	5	
	9	10	11	12
16	15	14	13	

- (A) A (B) B (C) C (D) D (E) E

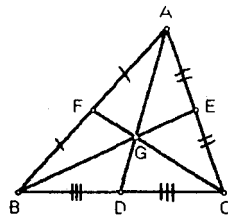
27. Emmy Noether was a German mathematician known for her groundbreaking contributions to:

- (A) number theory (B) calculus (C) Euclidean geometry
 (D) irrational numbers (E) abstract algebra

28. The current record of the Millersview Swatters baseball team is 35 wins and 15 losses. They have 40 more games to play. What will their overall winning percentage be if they win 5 games out of every 8 games they have left to play?

- (A) $66\frac{2}{3}\%$ (B) 70% (C) $68\frac{2}{11}\%$ (D) $62\frac{1}{2}\%$ (E) 80%

29. Point G is called the _____ of $\triangle ABC$.



- (A) incenter (B) circumcenter (C) centroid (D) orthocenter (E) cevian

30. Approximate to the nearest tenth: $(\log_3 5)(\log_5 7)(\log_7 9)(\log_9 11)$

- (A) 2.6 (B) 2.2 (C) 1.8 (D) 1.4 (E) 1.1

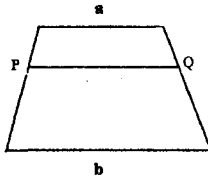
31. Which of the following is a reference angle for 2009° ?

- (A) 29° (B) 71° (C) 80° (D) 151° (E) 209°

32. How many points of intersection occur when $r = 3\sin(2\theta)$ and $r = 3\cos(2\theta)$ are graphed on a polar coordinate system?

- (A) 5 (B) 8 (C) 9 (D) 10 (E) 12

33. The isosceles trapezoid shown has a top base of length a and a bottom base of length b . The segment PQ divides the trapezoid into two similar trapezoids. The length of segment PQ is the _____ mean of a and b .



- (A) arithmetic (B) geometric (C) harmonic (D) quadratic (E) cevian

34. The sum of the finite geometric sequence, $10, -20, 40, -80, \dots, n$ is -3410 . How many terms are there in the sequence?

- (A) 30 (B) 25 (C) 20 (D) 15 (E) 10

35. Evaluate: $\int_{-n}^n (x^{-3} - 2x + 1) dx$

- (A) $-2n(n + 1)$ (B) $2n$ (C) $\frac{1}{2}n^2$ (D) $2n(n - 1)$ (E) n^{-2}

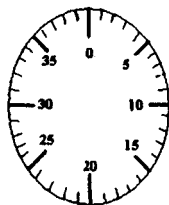
36. If $f''(x) = 30x$ and $f'(1) = 7$ and $f(-1) = 6$, then $f(0) =$ _____.

- (A) -5 (B) 0 (C) 3 (D) 5 (E) 9

37. $44_5 + 333_4 \times 2_3 - 1111_2 =$ _____ $_6$

- (A) 4321 (B) 135 (C) 423 (D) 55 (E) 343

38. The combination padlock shown below can be opened by turning right two or more whole turns and stopping at P. Then turn left one whole turn past P and stop at Q. Then turn right and stop at R. How many combinations exist if P is a multiple of 7, Q is divisible by 11, and R is a power of 2? Stopping at 0 is not allowed.

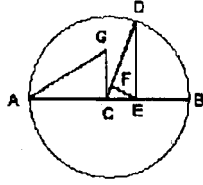


- (A) 90 (B) 100 (C) 75 (D) 120 (E) 14

39. The discriminant of $(5x - 3)(x - 3) = 0$ is _____.

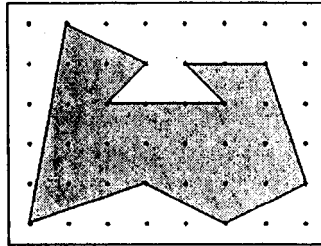
- (A) 106 (B) 135 (C) 144 (D) 279 (E) 504

40. Let AB be the diameter of the circle with center C with $CG \perp AB$, $DE \perp AB$, and $EF \perp DC$. Let $AE = x$ and $BE = y$ such that the arithmetic mean of x and y is 12.5 and the geometric mean of x and y is 12. Find CE .

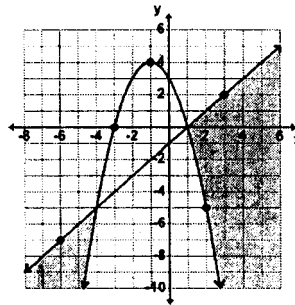


- (A) 12.98 (B) 11.52 (C) 7 (D) 6.25 (E) 3.5
41. Phil Upp's car gets 25 miles per gallon of gas when he drives a constant rate of 60 mph. The car gets 20 miles per gallon when he drives a constant rate of 70 mph. He takes a 250 mile trip and the cost of gas is \$1.85 per gallon. How much would he save by keeping a constant rate of 60 mph instead of 70 mph?
- (A) \$3.70 (B) \$4.63 (C) \$4.75 (D) \$5.00 (E) \$5.55
42. The function $f(x) = 6x^2 - 5x - 6$ crosses the x -axis at two points. Find the distance between the two points.
- (A) $\frac{5}{6}$ (B) $1\frac{5}{6}$ (C) $2\frac{1}{6}$ (D) 1 (E) $1\frac{1}{5}$
43. How many ordered pairs (x, y) are solutions to the equation $3x - 5y < 15$, where x, y are integers and $x \geq 0$ and $y \leq 0$?
- (A) 15 (B) 12 (C) 11 (D) 10 (E) 6
44. Simplify: $\left(\frac{2x^3 - 2xy^2 + x^2y - y^3}{2x^2 + 3xy - 2y^2}\right) \div \left(\frac{x^2 - y^2}{2y^2 - 3yx - 2x^2}\right)$
- (A) $2x^2 - xy - y^2$ (B) $2x + y$ (C) $x^2 - y^2$ (D) $-2x - y$ (E) $(2x - y)^{-1}$
45. Interstate 77 runs West to East. Radio station RUOK is 5 miles due North of I77. The broadcast range of the station is 9 miles. What length of the highway is in range of RUOK? (nearest tenth)
- (A) 14.5 miles (B) 15.0 miles (C) 16.1 miles (D) 19.7 miles (E) 20.6 miles
46. The circles $(x - \frac{1}{2})^2 + (y + 1)^2 = 10$ and $(x + 1)^2 + (y - 2)^2 = 5$ intersect in two points. The y -intercept of the line through the two points of intersection is (x, y) . Find y .
- (A) -2 (B) $-\frac{1}{2}$ (C) $\frac{3}{5}$ (D) $1\frac{11}{24}$ (E) $1\frac{1}{2}$
47. Find the multiplicative inverse, if it exists, of $\begin{bmatrix} 8 & 2 \\ 16 & 4 \end{bmatrix}$.
- (A) $\begin{bmatrix} -4 & 2 \\ 16 & -8 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & -16 \\ -2 & 8 \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{4} & \frac{1}{16} \\ \frac{1}{2} & \frac{1}{8} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{1}{8} & -\frac{1}{16} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$ (E) does not exist

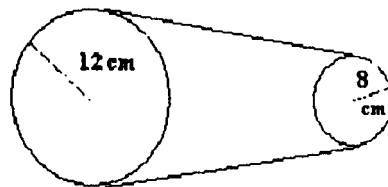
48. The adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown.



- (A) 20.5 cm^2 (B) 19 cm^2 (C) 13.5 cm^2 (D) 22 cm^2 (E) 14 cm^2
49. If $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, ..., $F_n = F_{n-2} + F_{n-1}$ and $L_0 = 2$, $L_1 = 1$, $L_2 = 3$, ..., $L_n = L_{n-2} + L_{n-1}$, for $n \geq 3$, then $F_{n+6} + F_{n+8} = ?$
- (A) L_{n+2} (B) L_{n+14} (C) $L_{n+7} + L_{n+9}$ (D) L_{n+7} (E) $L_{n+5} + L_{n+7}$
50. Which of the following system of inequalities is best represented by the shaded region?



- (A) $x - y \geq 1$
 $x^2 + 2x + y \geq 3$
- (B) $x + y \leq 1$
 $x^2 + 2x - y \geq 3$
- (C) $x - y \geq 1$
 $x^2 + 2x + y \leq 3$
- (D) $x + y \leq -1$
 $x^2 - 2x - y \leq 3$
- (E) $x - y \geq -1$
 $x^2 + 2x + y \geq 3$
51. A belt joins the two pulleys shown. The smaller pulley with the given radius revolves at the rate of 60 rpm. How much greater is the angular velocity of the smaller pulley than the larger pulley? (nearest tenth)



- (A) $4\pi \frac{\text{rad}}{\text{min}}$ (B) $8\pi \frac{\text{rad}}{\text{min}}$ (C) $20\pi \frac{\text{rad}}{\text{min}}$ (D) $40\pi \frac{\text{rad}}{\text{min}}$ (E) $60\pi \frac{\text{rad}}{\text{min}}$
52. An altitude of $\triangle LEG$ forms two right triangles, $\triangle LET$ and $\triangle GET$, such that $GT = 6''$, $LT = 4''$, and $m\angle EGT = 50^\circ$. Find $m\angle ELG$. (nearest degree)
- (A) 29° (B) 45° (C) 61° (D) 69° (E) 130°

53. Which of the following is a false statement?

- (A) $\cos \theta = \sin \left(\frac{\pi}{2} - \theta \right)$ (B) $\cos 3\theta = 4\cos^3\theta - 3\cos \theta$ (C) $\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$
 (D) $3\sin \theta = 4\sin^3\theta + \sin 3\theta$ (E) $\sin \theta = \cos (2\pi - \theta)$

54. Ura Gittenbigger paid \$12.00 for 6 hot dogs and 4 sodas at the county fair for lunch. Her niece, Ima Gittenbigtu paid \$9.00 for 5 hot dogs and 2 sodas. How much would it cost their cousin, I. B. Hungree, to buy 8 hot dogs and 5 sodas?

- (A) \$14.25 (B) \$14.75 (C) \$15.00 (D) \$15.75 (E) \$16.50

55. Max E. Mumm paid \$2.50 per square foot for a rectangular sheet of metal that is 4' by 8'. He made a rectangular water trough out of the sheet that would hold the greatest amount of water by cutting out the same size square from each of the four corners, folding up the remaining sides, and soldering the sides. What was the cost of metal used just to form the tank?

- (A) \$52.42 (B) \$58.57 (C) \$72.85 (D) \$77.14 (E) \$78.21

56. Let $f(x) = \sin x \cos x$. Find $f'(\frac{\pi}{6})$. (nearest hundredth)

- (A) 0.01 (B) 0.05 (C) 0.50 (D) 1.00 (E) undefined

57. Three fair six-sided dice are tossed and the spots on the top faces are recorded. One die is red, one is white, and one is blue. Which of the following has the greatest probability of occurring?

- (A) All three numbers are odd. (B) The red die is a 6 and the blue die is even.
 (C) All three numbers are prime (D) The white die is even and the red die is less than 3.
 (E) The red die is a 1, the white die is a 3, and the blue die is a 5.

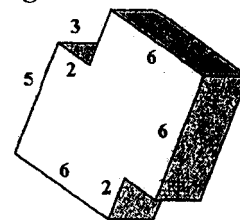
58. The odds of scoring less than 118 on this test is $\frac{7}{20}$. Based on the odds, if 297 students take this test how many would be expected to score 118 or greater?

- (A) 269 (B) 230 (C) 220 (D) 193 (E) 179

59. Let a, b, and c be positive integers where a, b, and c are not necessarily distinct. How many ordered triples (a, b, c) exist such that $a + b + c = 11$?

- (A) 33 (B) 45 (C) 55 (D) 66 (E) 90

60. Find the lateral surface area of this prism. All angles are right angles.



- (A) 81 units² (B) 87 units² (C) 93 units² (D) 96 units² (E) 108 units²

**University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2009
Answer Key**

- | | | |
|-------|-------|-------|
| 1. A | 21. E | 41. B |
| 2. B | 22. B | 42. C |
| 3. D | 23. C | 43. C |
| 4. E | 24. B | 44. D |
| 5. D | 25. C | 45. B |
| 6. B | 26. C | 46. D |
| 7. E | 27. E | 47. E |
| 8. A | 28. A | 48. A |
| 9. D | 29. C | 49. D |
| 10. C | 30. B | 50. A |
| 11. D | 31. A | 51. D |
| 12. C | 32. C | 52. C |
| 13. D | 33. B | 53. E |
| 14. D | 34. E | 54. D |
| 15. D | 35. B | 55. C |
| 16. B | 36. C | 56. C |
| 17. C | 37. E | 57. D |
| 18. B | 38. A | 58. C |
| 19. A | 39. C | 59. B |
| 20. E | 40. E | 60. D |