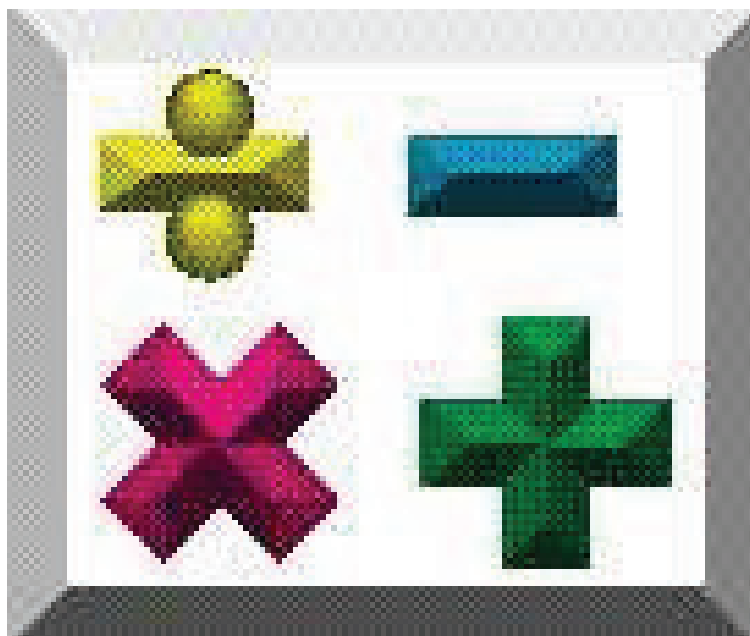




UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2010



**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

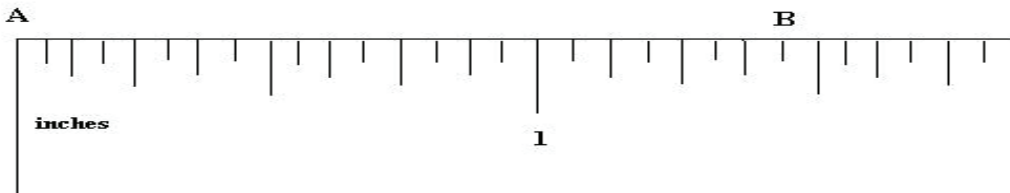
1. Evaluate: $30 - 24 \div 18 \times 12 + 6$

- (A) 6 (B) 10 (C) 20 (D) 35.888... (E) 350

2. Reid Moore went to the Ye Olde Book store to buy 3 copies of the same book for gifts. The regular price of the book is \$19.95. Because he is buying 3 copies he gets 25% off of the regular price of the second copy and 40% off the regular price of the third copy. What would the total cost of the 3 books be before taxes? (to the nearest cent)

- (A) \$ 26.93 (B) \$ 32.92 (C) \$ 38.91 (D) \$ 40.40 (E) \$ 46.88

3. Using the partial ruler shown below, find the distance from A to B.



- (A) $1\frac{1}{8}$ " (B) $1\frac{1}{4}$ " (C) $1\frac{3}{8}$ " (D) $1\frac{7}{16}$ " (E) $1\frac{1}{2}$ "

4. Which of the following is not a solution to $|8x - 6| - 4 \geq 2$?

- (A) $-2\frac{1}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $1\frac{4}{5}$ (E) 2

5. The function $f(x) = x^2 - x - 12$ crosses the x-axis at two points. Find the distance between the two points.

- (A) 8 (B) 7 (C) 6 (D) 4 (E) 1

6. A male zebra fish has 8 stripes. A female zebra fish has 7 stripes. What is the ratio of male fish to female fish, if the total number of stripes on all of the zebra fish in an aquarium totals 87?

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{7}{8}$ (D) $\frac{8}{7}$ (E) $\frac{3}{1}$

7. A box contains four rods whose lengths are 2", 3", 5", and 7". How many different triangles can be made using only three rods at a time.

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. A right cylinder water tank is 6 feet high and has an inside radius of 3 feet. The amount of water in the tank is 75% of its maximum capacity. How much water is in the tank? (nearest gallon)

- (A) 1270 gal (B) 635 gal (C) 734 gal (D) 317 gal (E) 952 gal

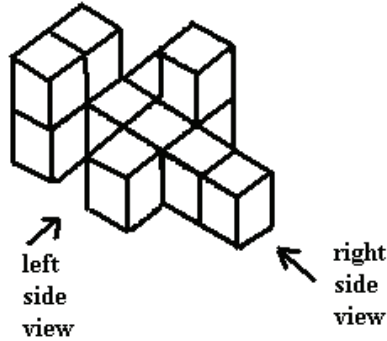
9. The region bounded by two radii of a circle and their intercepted arc is called a:

- (A) slice of pi (B) semicircle (C) secant (D) sector (E) segment

10. Noah Sense has 28 coins consisting of pennies, nickels, and quarters. He has four times as many nickels as pennies and half as many quarters as nickels. How much money does he have?

- (A) \$.86 (B) \$ 2.36 (C) \$ 2.84 (D) \$ 1.88 (E) \$.78

11. One-centimeter cubes are glued together to form the object in the figure shown. The two-dimensional perspective of the top view of this figure has a perimeter of:



- (A) 30 cm (B) 18 cm (C) 16 cm (D) 15 cm (E) 12 cm

12. If $8^{(k-1)} = 16^{(3k)}$, then $4^{(k-1)} = ?$

- (A) 256 (B) $\frac{1}{64}$ (C) 2 (D) $\sqrt[3]{4}$ (E) $1\frac{1}{3}$

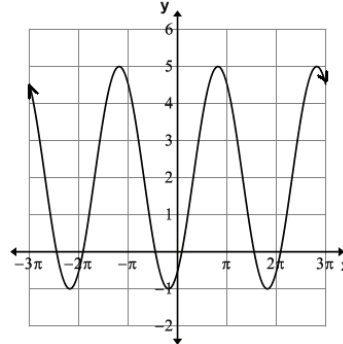
13. Babe, Dizzy, and Yogi are playing "toss and catch" with a baseball. The bearing from Babe to Dizzy is 254° . The bearing from Yogi to Dizzy is 344° . The bearing from Yogi to Babe is 32° . The distance from Yogi to Dizzy is 20 feet. How far is it from Yogi to Babe? (nearest inch)

- (A) 29' 11" (B) 28' 3" (C) 26' 11" (D) 22' 3" (E) 18' 1"

14. Determine the frequency of $y = 2 - 3 \cos 10x$. (nearest tenth)

- (A) 0.1 (B) 0.6 (C) 1.6 (D) 5 (E) 15.7

15. The equation $y =$ _____ will produce this graph.



- (A) $2 + 3\sin(x - 1)$ (B) $3 + 2\cos(x - 1)$ (C) $2 - 3\sin(x + 1)$
 (D) $2 - 3\cos(x - 1)$ (E) $2 + 3\sin(x + 1)$

16. Which of the following is a reference angle for 456° ?

- (A) 6° (B) 16° (C) 44° (D) 84° (E) 89°

17. Find the determinant of the 2×2 matrix $\begin{bmatrix} -2 & 3 \\ 5 & -7 \end{bmatrix}$.

- (A) 29 (B) 15 (C) 14 (D) 1 (E) -1

18. Given the arithmetic sequence 15, a, b, c, 47, ..., find $a + b + c$.

- (A) 24 (B) 32 (C) 78 (D) 93 (E) 108

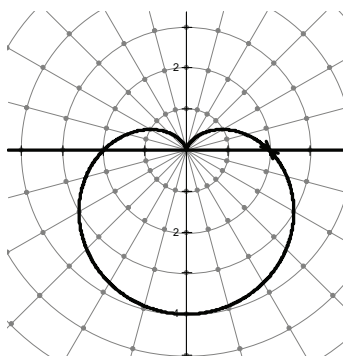
19. In the binomial expansion of $(3x - 1)^5$, the coefficient of the fourth term is:

- (A) -90 (B) -45 (C) -10 (D) 30 (E) 270

20. Find an equation of the line tangent to the curve $y = x^3 - 2x^2$ at the point $(1, -1)$.

- (A) $y = -x$ (B) $y = 6x - 4$ (C) $y = -\frac{2x}{3}$ (D) $y = 3$ (E) $y = 2 - x$

21. Which of the following polar equations will produce this graph on a polar grid?



- (A) $r = 2\sin \theta - 2$ (B) $r = 4 + 2\sin \theta$ (C) $r = 2 - 2\sin \theta$

- (D) $r = 2 - 4\cos \theta$ (E) $r = 2 + 2\cos \theta$

22. The area (in square units) of the region bounded by $y = -x^2 - 4x$ and $y = 0$ is:

- (A) $9\frac{1}{4}$ (B) $9\frac{5}{6}$ (C) $10\frac{1}{2}$ (D) $10\frac{2}{3}$ (E) $10\frac{3}{4}$

23. A box contains circular poker chips that are congruent in shape but not color. There are red ones, white ones, and blue ones. Drew Goode randomly draws out a chip. He gets 5 points if it is a blue one, 1 point for a white one, and he loses 3 points for a red one. The probability of drawing out a red one is 25%, a blue one is 60%, and a white one is 15%. What is his mathematical expectation on any one draw?

- (A) 5.0 (B) 3.9 (C) 3.0 (D) 2.4 (E) 2.1

24. What are the odds that a factor of 2010 is a prime number?

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) 1

25. The number of integers that satisfy the inequality $\frac{4}{15} \leq \frac{n}{5} \leq 1\frac{1}{30}$ is:

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

26. Simplify: $\frac{(n+1)! - (n-1)!}{(n-2)!}$

- (A) $n^2 + 1$ (B) $n^3 + 2n^2 - 2n + 1$ (C) $\frac{2(n+1)}{n-2}$ (D) $\frac{2}{n(n-1)}$ (E) $n^3 - 2n + 1$

27. The formula $e^{ix} = \cos x + i\sin x$, where e is the base of the natural logarithm and i is the imaginary unit, is named after:

- (A) Rene Descartes (B) Claudius Ptolemy (C) Theano of Crotona
(D) Leonard Euler (E) Eratosthenes of Cyrene

28. The odd numbers from 1 to 17 are to be placed in this magic square in which the rows and columns have the same sum. Find the value of x .

	1	
5		13
x		

- (A) 3 (B) 7 (C) 9 (D) 11 (E) 15

29. $P = \{p,l,u,s\}$, $Q = \{m,i,n,u,s\}$, and $R = \{t,i,m,e,s\}$. How many elements are in $(P \cup Q) \cap (P \cup R)$?

- (A) 10 (B) 6 (C) 5 (D) 4 (E) 2

30. The number 12010 in base 3 is equivalent to the number $wxyz$ in base 5, where w , x , y , and z are digits. Find $w + x + y + z$.

- (A) 10 (B) 9 (C) 8 (D) 6 (E) 3

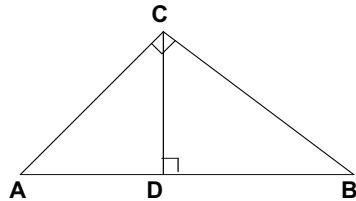
31. Simplify: $a^5 \div b^{-4} \times a^{-4} \times b^5 \div a^3 \times b^{-3}$

- (A) $a^{-2} b^6$ (B) $a^4 b^{-2}$ (C) $a^{-2} b^{12}$ (D) $a^4 b^2$ (E) $a^2 b^{-6}$

32. Simplify: $\frac{x^2 - 9}{4x + 12} \div \frac{x^2 - x - 6}{x^2 + 2x}$

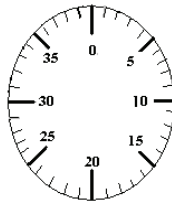
- (A) $\frac{x}{4}$ (B) $\frac{4}{x+3}$ (C) $\frac{x-3}{4(x+3)}$ (D) $\frac{4}{x}$ (E) $\frac{x+3}{4}$

33. The distance from Abilene to Dallas by way of I30 is 185 miles. Ima Slow is leaving Abilene on I30 at 9:00 a.m. driving toward Dallas at 55 mph. Ura Quick is leaving Dallas on I30 at 9:00 a.m. driving toward Abilene at 70 mph. What time will they meet? (nearest minute)
- (A) 10:48 a.m. (B) 10:41 a.m. (C) 10:29 a.m. (D) 10:19 a.m. (E) 10:10 a.m.
34. $\angle A$ and $\angle B$ are complementary angles. $\angle A$ and $\angle C$ are supplementary angles. Find $m\angle C$ if $m\angle A = 2x - 5$ and $m\angle B = x + 2$.
- (A) 121° (B) 149° (C) 135° (D) 123° (E) 147°
35. If $a_1 = 2$, $a_2 = 4.5$, and $a_3 = 7$ are the first 3 terms of an arithmetic sequence, then $a_9 = ?$
- (A) 17 (B) 19.5 (C) 21 (D) 22 (E) 24.5
36. The graph of $4x^2 + 9y^2 - 16x + 18y = 2$ is a(n):
- (A) parabola (B) line (C) hyperbola (D) ellipse (E) circle
37. The eccentricity of the hyperbola $4x^2 - y^2 = 4$ is:
- (A) $\sqrt{5}$ (B) $\frac{\sqrt{17}}{2}$ (C) $\sqrt{15}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{\sqrt{5}}{2}$
38. If $\cos \theta < 0$ and $\tan \theta > 0$ which quadrant will θ terminate in?
- (A) QI or QII (B) QI only (C) QII only (D) QIII only (E) QII or QIII
39. Let $\|V_1\| = 15$ and $\|V_2\| = 9$, where the direction angles of V_1 and V_2 are 20° and 80° , respectively. Find $\|V_1 + V_2\|$. (nearest tenth)
- (A) 23.6 (B) 17.5 (C) 20.7 (D) 12.0 (E) 21.0
40. Find AD if AB = 90 cm. and AC = 50 cm. (nearest cm)



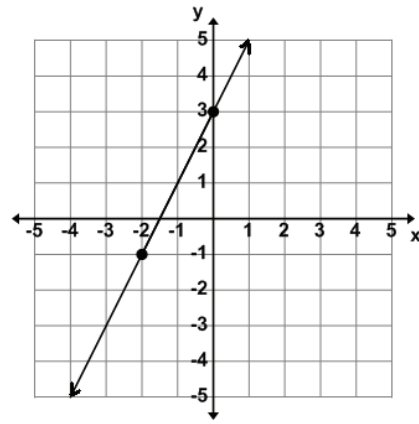
- (A) 67 cm (B) 19 cm (C) 28 cm (D) 60 cm (E) 45 cm
41. $\int (-x \sin x) dx = \underline{\hspace{2cm}} + C$, where C is some arbitrary constant.
- (A) $\cos x^2$ (B) $x \cos x - \sin x$ (C) $-\sin x \cos x$ (D) $-x \cos x$ (E) $x \sin x - \cos x$

42. If $f''(x) = 6$ and $f'(-1) = -8$ and $f(1) = 2$, then $f(-2) = \underline{\hspace{2cm}}$.
- (A) 20 (B) 17 (C) 8 (D) -7 (E) -14
43. Find the instantaneous rate of change of the reciprocal of a number with respect to the number when the number is 4.
- (A) $-\frac{1}{16}$ (B) $-\frac{1}{4}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{1}{16}$
44. How many different letter arrangements can be made by rearranging the letters in the word 'LETTER'?
- (A) 180 (B) 21 (C) 120 (D) 24 (E) 360
45. Willie Lawkit can't remember the combination to the padlock shown. He knows that the first number is greater than 30, the second number is a positive Fibonacci number, and the third number is a factor of 30. How many combinations can he try to open the lock?



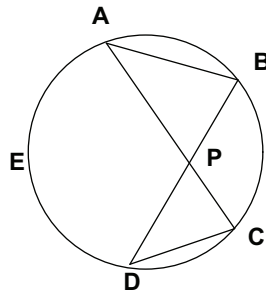
- (A) 25 (B) 378 (C) 576 (D) 72 (E) 480
46. The operation " \triangle " is defined by: $a \triangle b = a^b - b^a$. What is the value of $(0 \triangle 1) \triangle (2 \triangle 3)$?
- (A) -1 (B) 0 (C) 1 (D) 2 (E) 4
47. $3(x + 4) = 5$ and $3(4 + x) = 5$ is an example of the _____ property.
- (A) reflexive (B) commutative (C) identity (D) associative (E) distributive
48. Slim Sails rents kayaks and life vests for white water rafting. The kayak rental fee last year was \$40 and the life vest rental fee last year was \$12. This year, the kayak rental fee increased 15% and the life vest fee decreased 25%. What is the overall percent increase in rental fees for the kayak and vest from last year to this year? (nearest tenth)
- (A) 10.0% (B) 9.1% (C) 8.3% (D) 6.5% (E) 5.8%
49. If $-3(2 - x) = 2(x + 3)$ then $(2x - 3)$ equals:
- (A) 12 (B) -9 (C) 21 (D) -3.4 (E) 1.8
50. The area of a right isosceles triangle is 12.5 cm^2 . Its perimeter is: (nearest tenth).
- (A) 18.7 cm (B) 11.4 cm (C) 21.2 cm (D) 11.7 cm (E) 17.1 cm

51. Find the slope of a line perpendicular to the line drawn in the graph below.



- (A) -2 (B) -1.5 (C) $-.5$ (D) $.5$ (E) 2

52. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O and point E lies on circle O. Which of following is a true statement?



- (A) $m\angle ABD = \frac{1}{2} \times m\widehat{AED}$ (B) $m\angle BPC = \frac{1}{2} \times m\widehat{CB}$ (C) $m\angle ACD = 2 \times m\widehat{AED}$
 (D) $m\angle APD = m\angle ABP + m\angle DCP$ (E) $m\angle ABP + m\angle BDC$

53. A regular polygon has S sides and D diagonals. If the polygon had one more side, $S + 1$, it would have $D + 10$ diagonals. The polygon is a:

- (A) octagon (B) nonagon (C) decagon (D) undecagon (E) dodecagon

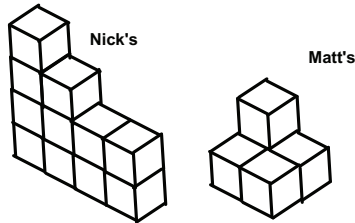
54. Let $f(x) = 2 - 5x$ and $g(x) = 3x + 5$. If $h(x)$ is the inverse function of $\frac{f(x)}{g(x)}$, then $h(-4) = ?$

- (A) $-\frac{22}{7}$ (B) $-\frac{18}{17}$ (C) $\frac{7}{22}$ (D) $\frac{17}{18}$ (E) 4

55. $\sin \theta \sec \theta + \cos \theta \csc \theta$ is equivalent to:

- (A) $\frac{\csc \theta}{\cot \theta}$ (B) $\frac{\tan^2 \theta}{\csc \theta}$ (C) $\tan^2 \theta + 1$ (D) $\frac{\sec^2 \theta}{\tan \theta}$ (E) $\sec^2 \theta - 1$

56. Willie Ketchit drops a golfball from a height of 10 meters. Each time it hits the ground it rebounds to a height of 50% of the distance it fell. Find the total distance the golfball travels when it reaches the ground the third time. (nearest tenth)
- (A) 35.0 m (B) 32.5 m (C) 30.0 m (D) 28.5 m (E) 25.0 m
57. The polynomial $2x^4 - 8x^2 + x + 5$ has at most ____ negative zeros.
- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
58. Coach Winters has 4 seniors, 5 juniors, 3 sophomores, and 4 freshmen on her math team. How many ways can she form practice groups of four members consisting of one member from each of the grade levels?
- (A) 16 (B) 81 (C) 108 (D) 240 (E) 256
59. Romeo, Juliet, and three classmates are randomly assigned seats in a row of five chairs. What is the probability that Romeo and Juliet will be seated next to each other?
- (A) 20% (B) 25% (C) 30% (D) 35% (E) 40%
60. Matt and Nick constructed two buildings using identical cubes. Matt's building weighs 200 g, and Nick's building weighs 600 g. How many of the cubes in Nick's building are hidden and cannot be seen in the figure?



- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation A • 2010
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. C | 41. B |
| 2. E | 22. D | 42. B |
| 3. D | 23. D | 43. A |
| 4. C | 24. B | 44. A |
| 5. B | 25. B | 45. C |
| 6. A | 26. E | 46. B |
| 7. B | 27. D | 47. B |
| 8. E | 28. B | 48. E |
| 9. D | 29. B | 49. C |
| 10. C | 30. D | 50. E |
| 11. B | 31. A | 51. C |
| 12. B | 32. A | 52. A |
| 13. A | 33. C | 53. D |
| 14. B | 34. D | 54. A |
| 15. A | 35. D | 55. D |
| 16. D | 36. D | 56. E |
| 17. E | 37. A | 57. C |
| 18. D | 38. D | 58. D |
| 19. A | 39. E | 59. E |
| 20. A | 40. C | 60. D |