UIL, PSIA, AND TMSCA NUMBER SENSE MAGIC

The following are excerpts from UIL, PSIA, and TMSCA Number Sense Magic workbook. The workbook has several examples for each shortcut. At the end of the workbook is a section that contains information that students should memorize.

Multiplication by 11

Step #1 : The first digit (units digit) of number being multiplied by 11 is the first digit of the answer.

Step #2 : The sum of the tens digit and the units digit of the number will give the tens digit of the answer. If the sum is greater than or equal to 10 make sure to carryover to Step #3.

Step #3 : Repeat addition of consecutive digits (tens to hundreds, hundreds to thousands, etc.) until the final digit of the number being multiplied by 11 is reached. If necessary, add carryovers from previous steps.

Step #4 : This final digit (plus any carryover) will be the final digit of the answer.

Example A : 35 x 11 = __________

Solution :

Step #1 : The first digit of the answer is 5.
Step #2 : 3 + 5 = 8
Step #3 : The final digit is 3.

Answer : 385

Double and half method of multiplication

Although this shortcut can be used in other situations, I recommend that it be used when multiplying a number that ends with five by an even number.

Step #1 : Double the number ending in 5.
Step #2 : Take half of the even number.
Step #3 : Find the product of the results from Step #1 and Step #2.

Example A : 15 x 12 = _________.

Step #1 : Double the 15. The result is 30.
Step #2 : Take one half of 12. The result is 6.
Step #3 : 30 x 6 = 180

Multiplying two numbers close to 100 (Both numbers are less than 100)

Step # 1 : Find the difference of each number and 100. Multiply the result. This product will give the first two digits of the answer (the tens and units digits). If the product is greater than one hundred, carry the hundreds digit to Step #2.

Step # 2 : Subtract the difference of one of the numbers and one hundred from the other number. Add any carryover from Step #1. This result will be the remaining digits of the answer.

Example A : $95 \times 98 =$ ________

Step #1 : $(100 - 95)(100 - 98) = 5(2) = 10$
Step #2 : $95 - (100 - 98) = 95 - 2 = 93$

Answer : 9310

Multiplying two numbers whose units digits add up to 10 and whose other digits are the same

Step #1 : Multiply the units digits. This should give the first two digits of the answer. If the product consists of one digit, make sure to write a 0 in front of it.
Step #2 : Increase one of the digits preceding the units digit by 1 and multiplying by the other digit preceding the units digit. This should give the remaining digits of the answer.

Example A : $43 \times 47 =$ __________.

Step #1 : $3 \times 7 = 21$
Step #2 : $(4 + 1)(4) = 5(4) = 20$

Answer : 2021

Multiplying by 12

The key to this shortcut is to always remember to double and add to the right. You continue doing this until you get to the last digit (the left side of the number). At this point you imagine doubling 0 and add this last digit (plus any carryover).

Exaple A : $63 \times 12 =$ _________.

Step #1 : Double the 3 and add to the right. The result is 6.
Step #2 : Double the 6 and add to the right (add it to the 3). The Result is 15. Write down the 5 and carryover the 1.
Step #3 : Double 0 and add 6 plus the carryover from Step #2. The Result is 7.

Answer : 756

Multiplying by 15

Step #1 : Find one-half of the number being multiplied by 15.
Step #2 : Add the result from Step #1 to the number being multiplied by 15.
Step #3 : Add a 0 to the result from Step #2.

Example A : $15 \times 42 =$ __________.

Step #1 : $\frac{1}{2}(42) = 21$

Step #2 : 21 + 42 = 63
Step #3 : 630

Answer : 630

Find the remainder when dividing by 11

Step #1 : Beginning with the units digit, add every other digit from left to right.
Step #2 : Find the sum of the remaining digits
Step #3 : Subtract Step #2 from Step #1

(i) If the result is less than 11, this is the remainder.

Example A : 2537 ÷ 11 has a remainder of __________.

Step #1 : 7 + 5 = 12
Step #2 : 3 + 2 = 5
Step #3 : 12 - 5 = 7

The remainder is 7.

(ii) If the result is negative, keep adding 11 until the result is positive. This is the remainder.

Example A : 4782 ÷ 11 has a remainder of __________.

Step #1 : 2 + 7 = 9
Step #2 : 8 + 4 = 12
Step #3 : 9 - 12 = -3 ; -3 + 11 = 8

The remainder is 8,

(iii) If the result is greater than 11, keep subtracting 11 until the result is less than 11.

Example A : 2819 ÷ 11 has a remainder of __________.

Step #1 : 9 + 8 = 17
Step #2 : 1 + 2 = 3
Step #3 : 17 - 3 = 14 ; 14 - 11 = 3

The remainder is 3.

(iv) If the result is 11, than the remainder is 0.

Example A : 2849 ÷ 11 has a remainder of __________.

(9 + 8) - (4 + 2) = 17 - 6 = 11 ;
The remainder is 0.

REDUCING COMMON FRACTIONS BY USING KNOWLEDGE OF DIVISIBILITY RULES

The use of divisibility rules when reducing common fractions is implied in math textbooks. Students should first become familiar with the following rules for divisibility.

Divisibility by 2 : the number is even (it ends with 0, 2, 4, 6, or 8)

Divisibility by 3 : the sum of the digits is divisible by 3

Divisibility by 4 : the last two digits is divisible by 4

Divisibility by 5 : the number ends in 0 or 5

Divisibility by 6 : the number is even and the sum of the digits is divisible by 3

Divisibility by 8 : the last three digits are divisible by 8

Divisibility by 9 : the sum of the digits is divisible by 9

Divisibility by 10 : the number ends in 0

Divisibility by 11 : Add every other digit beginning with the units digit. Subtract the Sum of the remaining digits. If the result is 0, the number is divisible by 11.

FINDING THE LCM OF TWO NUMBERS

The following method of finding the LCM of two numbers has not appeared in a mathematics book that I am aware of, yet I am confident that it is less confusing then
the method that makes use of "prime factorization". Beginning with its first introduction to students, the following method should be an integral part of every mathematics book.

Step #1 : Students should be made aware that the product of the GCF and LCM of two numbers is equal to the product of the two numbers. Examples should be provided to make this obvious.

Step #2 : Using the statement made in Step #1, it should be noted that the following is true :

LCM = (product of numbers) ÷ GCF

Example A : Find the LCM of 12 and 20.

Solution : LCM = (12 s 20) ÷ (GCF of 12 and 20)

= (12 x 20) ÷ 4

At this point, students should be told that the easiest way to solve the problem is to divide 4 into one of the two numbers and multiply the result by the other number.

Note : $(12 \div 4) \times 20 = 3 \times 20 = 60$

The following are problems that appear on either UIL, TMSCA or PSIA Number Sense tests. These problems range from 4th grade to 8th grade. Keep in mind that many of these problems are also featured on high school tests.

1. $702 - 207 =$ __________.

Note : This shortcut can be used when find the difference of two numbers whose digits are reversed.

Step #1 : Find the difference of the units and hundreds place.

$7 - 2 = 5$

Step #2 : Multiply the result from Step #1 by 99 (It is easier if you multiply by 100 - 1.

$5(100 - 1) = 500 - 5 = 495$

5. $72 - 8 \div 4 =$ _________.

When expressions have more than one operation, we have to follow rules for the order of operations.

(1) First do all operations that lie inside parentheses.
(2) Next, do any work with exponents or radicals.
(3) Working from left to right, do all multiplication and division.
(4) Finally, working from left to right, do all addition and subtraction.

Solution : $72 - (8 \div 4) = 72 - 2 = 70$

9. $32 + 16 + 8 + 4 + 2 + 1 =$ __________.

Note : These terms are powers of 2 beginning with 2^0.

Solution : $32(2) - 1 = 64 - 1 =$ _________.

Example A : $1 + 2 + 4 + 8 + 16 + 32 + 64 =$ _________.

Solution : $64(2) - 1 = 128 - 1 = 127$

12. $\frac{13}{40} =$ __________ (decimal).

Solution : Think of the fraction that you have (without the 0 in the denominator). Convert it into a mixed number and then a decimal fraction.

$$\frac{13}{4} = 3\frac{1}{4} = 3.25$$

Then move the decimal one place to the left to get the answer.

.325

Example A : $\frac{23}{40}$ = __________ (decimal)

Solution : $\frac{23}{4} = 5\frac{3}{4} = 5.75$; The answer is .575

18. $1 + 2 + 3 + 4 + \ldots + 19 =$ __________.

When adding the positive integers from 1 to n, the sum is equal to

$\frac{n(n+1)}{2}$.

Solution : $\frac{19(19+1)}{2} = 19\left(\frac{20}{2}\right) = 19(10) = 190$

19. $1 + 3 + 5 + \ldots + 19 =$ __________.

Rule : $1 + 3 + 5 + \ldots + k = \left(\frac{k+1}{2}\right)^2$

Solution : $\left(\frac{19+1}{2}\right)^2 = 10^2 = 100$

Example A : $1 + 3 + 5 + \ldots + 13 =$ __________.

Solution : $\left(\frac{13+1}{2}\right)^2 = 7^2 = 49$

24. $3\frac{1}{3}\%$ = __________ (fraction).

Memorization of percents is recommended to make this easier to solve.

Step #1 : Convert the mixed number into an improper fraction.

$$\frac{10}{3}$$

Step #2 : Add two zeroes to the denominator, then reduce to lowest terms.

$$\frac{10}{300} = \frac{1}{3}$$

27. If 6 pears cost \$1.21, then 2 dozen pears cost \$__________.

Note : 2 dozen pears = 24 pears. To find the cost of 24 pears, multiply the cost of 6 pears by 4.

\$1.21(4) = \$4.84

28. The additive inverse of - 6 is ___________.

The additive inverse of a number is the opposite of the number. A number plus its additive inverse is equal to 0.

Solution : The additive inverse of – 6 is :

$-(-6) = 6$

Example A : The additive inverse of $5\frac{2}{3}$ is __________.

Solution : The additive inverse of $5\frac{2}{3}$ is :

$-(5\frac{2}{3}) = -5\frac{2}{3}$

Example B : The additive inverse of 3.7 is ___________.

Solution : The additive inverse of 3.7 is :

$-(3.7) = -3.7$

30. $4\frac{1}{6} \times 12 =$ __________.

Solution : $4(12) + \frac{1}{6}(12) = 48 + 2 = 50$

33. CLVI = _________ (Arabic Numeral)

Note : M = 1000, D = 500, C = 100, L = 50, X = 10, V = 5 and I = 1

Solution : CLVI = 100 + 50 + 5 + 1 = 156

Example A : XLIII = __________ (Arabic Numeral).

Solution : (50 - 10) + 3 = 40 + 3 = 43

44. The smallest prime divisor of 119 is ________.

Prime numbers ; 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Solution : 119 = 7(17) ; Answer : 7

45. 56 has _________ positive prime divisors.

Solution : $56 = (8)(7) = (2^3)(7)$; 56 has 2 positive prime divisors.

Example A : 40 has __________ positive prime divisors.

Solution : $40 = (8)(5) = 2^3(5)$; 40 has 2 positive prime divisors.

48. The negative reciprocal of $-1\frac{1}{9}$ is __________.

Note : The negative reciprocal of a number is the opposite of the reciprocal of a number. If the number is positive, the negative reciprocal will be negative. If the number is negative, the negative reciprocal will be positive.

Solution : $-\left(-\frac{9}{10}\right) = \frac{9}{10}$

Example A : The negative reciprocal of $\frac{2}{5}$ is __________.

Solution : $-\left(\frac{5}{2}\right) = -\frac{5}{2}$

Example B : The negative reciprocal of -3.2 is __________.

Note : $-3.2 = -3\frac{2}{10} = -3\frac{1}{5} = -\frac{16}{5}$

Solution : $-\left(-\frac{5}{16}\right) = \frac{5}{16}$

49. The range of 1, 2, 1, 3, 1, 4, and 0 is __________.

The range of a list of numbers is the difference between the smallest and largest Numbers.

Solution : $4 - 0 = 4$

Example A : The range of 5, 9, 12, 23, 14, and 2 is __________.

Solution : $23 - 2 = 21$

56. Which is smaller, $\frac{8}{11}$ or $\frac{10}{13}$?

Solution : Cross multiply (from denominator to numerator). The smaller number will be the fraction with the smaller product.

$8(13) = 104$ (104 is associated with the fraction $\frac{8}{11}$.)

$11(10) = 110$ (110 is associated with the fraction $\frac{10}{13}$.)

Since 104 is less than 110, the smaller fraction is $\frac{8}{11}$.

Rule : To convert from miles per hour to feet per second multiply by $\frac{22}{15}$.

Solution : $15\left(\frac{22}{15}\right) = 22$

Example A : 30 miles per hour = ________ ft/sec.

Solution : $30\left(\frac{22}{15}\right) = 2(22) = 44$

68. What is the length of the base of a triangle with an area of 27 and an altitude of 6? ________.

Solution : Area = $\frac{(base)(height)}{2}$

$27 = \frac{(base)(6)}{2}$; 27(2) = (base)(6) ; 54 = (base)(6) ; base = $\frac{54}{6} = 9$

Step #1 : Double the area

27(2) = 54

Step #2 : Divide by the altitude.

$54 \div 6 = 9$

72. $6^3 \div 3^3$ = ________.

Solution : $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$

Example A : $12^3 \div 3^3$ = ________.

Solution : $\frac{12^3}{3^3} = \left(\frac{12}{3}\right)^3 = 4^3 = 64$

Example B : $36^2 \div 4^2$ = ________.

Solution : $\frac{36^2}{4^2} = \left(\frac{36}{4}\right)^2 = 9^2 = 81$

73. $(23 \times 5 + 4) \div 7$ has a remainder of ________.

Solution : The remainder when 23 is divided by 7 is 2. The remainder when 5 is Divided by 7 is 5. The remainder when 4 is divided by 7 is 4.

$(23 \times 5 + 4) \div 7$ has a remainder that can be found by first doing the following : $2 \times 5 + 4 = 14$, then find the remainder when 14 is divided by 7. The answer is 0.

Example A : $(32 \times 13 + 45) \div 6$ has a remainder of ________.

Solution : Find the remainder of each number inside of the parentheses.

The remainder when 32 is divided by 6 is 2. The remainder when 13 is divided by 6 is 1. The remainder when 45 is divided by 6 is 3,

Substitute the remainders for numbers inside the parentheses.

$2 \times 1 + 3 = 2 + 3 = 5$; The answer is 5.

75. The product of the GCF and the LCM of 24 and 30 is __________.

The product of the GCF and the LCM of any two numbers is equal to the product of the two numbers.

Solution : $24(30) = 720$

82. $8\frac{2}{3} \times 8\frac{1}{3} =$ __________ (mixed number)

Note : For this shortcut to work, the sum of the fractions is 1 and the whole numbers are the same.

Solution : Step #1 : Multiply the fractions.

$$\frac{2}{3}\left(\frac{1}{3}\right) = \frac{2}{9}$$

Step #2 : Find the product of one of the whole numbers and the whole number plus 1.

$$8(8 + 1) = 8(9) = 72$$

Answer : $72\frac{2}{9}$

83. $4\frac{1}{4} \times 16\frac{1}{4} =$ __________ (mixed number).

If the product of the fractional part of one of the mixed numbers and the sum of the whole numbers is equal to a whole number then do the following :

Step #1 : Multiply the fractions.

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

Step #2 : Multiply the fractional part of one mixed number and the sum of the whole numbers.

$$\frac{1}{4}(4+16) = 5$$

Step #3 : Add the result of Step #2 to the product of the whole numbers.

$$5 + 4(16) = 5 + 64 = 69$$

Answer : $69 \frac{1}{16}$

86. Approximate : 29014 ÷ 802 = ___________.

Solution : Think of the problem as being 29000 ÷ 800. Since $\frac{29}{8} = 3\frac{5}{8} =$ 3.625, the answer can be found. Keep in mind that 290 ÷ 80 is a 2-digit answer. Answer is 36

Range : 35 – 37

Example A : Approximate : 381023 ÷ 304 = _________.

Solution : Think of the problem as being 380000 ÷ 300. Since $\frac{38}{3} = 12\frac{2}{3} = 12.66\ldots$, the answer can be found by knowing that 3800 ÷ 3 is a 4-digit answer. Answer is 1266.

Range : 1191 - 1316

87. If A = {2, 8, 0, 5, 3} and B = {4, 3, 7, 9, 5}, then A ∩ B is {____________}.

Note : The intersection of two given sets, is a set whose elements are contained in both sets?

Solution : Since 5 and 3 are elements in both sets A and B, then the answer is {5, 3}.

89. If A = (2, 7, 0} and B = {1, 3}, then the Cartesians product of A and B has ________ elements.

Note : The Cartesian product of two sets is a set of ordered pairs where the left term comes from the First set given and the right term comes from the second set given.

Solution : The Cartesian product of sets A and B is a set whose elements are (2, 1), (2, 3), (7, 1), (7, 3), (0, 1) (0, 3). Thus, the answer is 6.

Rule : To find the number of elements in the Cartesian Product of two given sets, simply multiply the number of elements in each set.

3(2) = 6

90. 143 x 21 = _________.

Rule : When multiply a multiple of 7 by 143, simply divide the multiple of 7 by 7, then multiply by 1001.

Solution : 21 ÷ 7 = 3 ; 3(1001) = 3003

93. $5 - 10 + 15 - 20 + \ldots + 75 - 80 =$ ________.

Note : The sequence consists of multiples with alternating signs. If the last term is negative the answer is equal to the last digit divided by 2.

Solution : $(-80) \div 2 = -40$

100. 72% of 36 is 18% of ________.

If a% of b is c% of d, then $d = \frac{ab}{c}$.

Solution : $\frac{72(36)}{18} = 72\left(\frac{36}{18}\right) = 72(2) = 144$

Example A : 45% of 27 is 9% of ________.

Solution : $\frac{45(27)}{9} = 45\left(\frac{27}{9}\right) = 45(3) = 135$

101. The set {M, A, T, H} has ________ subsets.

Note : An n-element set has 2^n subsets.

Solution : $2^4 = 16$

102. How many elements in the power set of {3, 8, 1}? ________.

Note : The power set of a given set is a set whose elements are the subsets of the set.

Rule : 2^n, where n is the number of elements in the set

Solution : $2^3 = 8$

Example A : How many elements in the power set of a 5-element set? ________.

108. $\frac{7}{9} + \frac{9}{7} =$ ________ (mixed number).

Step #1 : The numerator of the answer is equal to the square of the difference of the numerator and denominator of one of the fractions.

$(9 - 7)^2 = 2^2 = 4$

Step #2 : The denominator of the answer is equal to the product of the denominators of the two given fractions.

$9(7) = 63$

Step #3 : If the fraction resulting from Step #1 and Step #2 is a proper fraction, then the whole number of the answer is 2. If the resulting fraction is improper, convert it into a mixed number and add 2 to this result.

Answer : $2 \frac{4}{63}$

109. $16 \times \frac{16}{19}$ = __________ (mixed number).

Step #1 : Find the difference between the numerator and the denominator of the fraction. Square this difference. The fractional part of the answer is this result over the denominator of the given fraction.

$$\left(\frac{(19-16)^2}{19}\right) = \frac{3^2}{19} = \frac{9}{19}$$

Step #2 : From the whole number in the given problem, subtract the difference of the numerator and denominator of the given fraction.

$16 - (19 - 16) = 16 - 3 = 13$

Answer : $13 \frac{9}{19}$

110. .414141... = __________ (proper fraction).

Write a fraction whose numerator is the repeated digits and whose denominator consists of as many 9's as there are repeating digits.

Solution : $\frac{41}{99}$

Example A : .666... = __________ (proper fraction).

Solution : $\frac{6}{9} = \frac{2}{3}$

113. $27^2 - 23^2$ = __________.

Note ; $a^2 - b^2 = (a + b)(a - b)$; The difference of squares is equal to the product of the sum of the two numbers and the difference of the two numbers.

Solution : $(27 + 23)(27 - 23) = 50(4) = 200$

114. The next term in the sequence 3, 5, 8, 13, 21, 34, ... is __________.

Note : This is a Fibonacci style sequence. The sum of the 1st and 2nd terms is equal to the third term. The sum of the 2nd and 3rd terms is equal to the 4th term. The sum of the 3rd and 4th terms is equal to the 5th term, and so on.

Solution : $21 + 34 = 55$

115. Approximate : 457689 ÷ 111 = __________.

Note : $.111... = \frac{1}{9}$. Think of this problem is being $460000 \div \frac{1}{9} \times 1000 =$

$\frac{460000(9)}{1000} = 460(9) = 4140$ (This is an approximate answer).

In general, when dividing by 111, multiply the first two digits by 9, then place the decimal.

Example A : 31143 ÷ 111 = __________.

Solution : 31 x 9 = 279 ; Answer is 279

116. Approximate : 719262 x 111 = __________.

Solution : Step #1 : $\frac{72}{9} = 8$

Step #2 : Quick guess : 700,000 x 100 = 70,000,000

Answer : 80,000

117. Approximate : 20 x 21 + 22 x 23 = __________.

Rule : Double the product of the first term and the last term. Interesting to note that the exact answer can be found by simply adding 6 to this result.

2(20)(23) = 2(460) = 920

Range : 880 - 972

121. The 13th term of 2, 5, 8, 11, 14, ... is __________.

Step #1 : Find what number is being added to find each term in the pattern.

They are adding 3.

Step #2 : Subtract 1 from the term you are looking for and multiply by the result of Step#1.

(13 - 1) x 3 = 12 x 3 = 36

Step#3 : Add the result of Step #2 to the first term in the patteren.

36 + 2 = 38

Example A : The 12th term of 3, 10, 17, 24, 31, ... is __________.

Step #1 : Note that they are adding 7 to find each term.

Step #2 : (12 - 1) x 7 = 11 x 7 = 77

Step #3 : 77 + 3 = 90

124. If $7x + 6 = 34$, then $x =$ _________.

Solution : $7x = 34 - 6$; $7x = 28$; $x = 4$

128. What is the area of a square with a diagonal of 8? ________.

Rule : The area of a square is equal to one-half the square of the diagonal.

Solution : $\frac{1}{2}(8^2) = \frac{1}{2}(64) = 32$

129. Approximate : $1428 \times 21 =$ _________.

Divide the number being multiplied by 1428 by 7, then move the decimal 4 places to the right.

$21 \div 7 = 3$; Answer : 30,000

Range : 28,489 - 31,487

133. The smallest leg of a right triangle with integral sides is 7". The hypotenuse is __________".

Note : The given leg is an odd prime number.

Step #1 : Square the number. $7^2 = 49$

Step #2 : Find two consecutive integers whose sum is equal to the result of Step #1.

$49 = 24 + 25$

Step #3 : The resulting Pythagorean Triplet is 7, 24, 25, The hypotenuse is 25.

Example A : The smallest leg of a right triangle with integral sides is 11". The longest leg is __________".

Solution : Step #1 : $11^2 = 121$
Step #2 : $121 = 60 + 61$
Step #3 : 11, 60, 61, The long leg is 60.

135. The sides of a right triangle are integers. If one leg is 6, then the other leg is ________.

If the given leg is even, do the following :

Step #1 : Divide the leg in half and square it.

$$\left(\frac{6}{2}\right)^2 = 3^2 = 9$$

Step #2 : Find the integers on either side of the result from Step #1.

8 and 10 are the integers on either side of 9.

Step #3 : Select the smaller integer obtained in Step #2.

Answer : 8

Example A : The sides of a right triangle are integers. If one leg is 10, then the other leg is ________.

Step #1 : $\left(\frac{10}{2}\right)^2 = 5^2 = 25$

Step #2 : 24 and 26 are the integers on either side of 25.

Step #3 : Answer : 24

137. $\frac{1}{12} + \frac{1}{20} + \frac{1}{30} =$ ________.

Note : $\frac{a}{b(b+1)} + \frac{a}{(b+1)(b+2)} + \frac{a}{(b+2)(b+3)} = \frac{3a}{b(b+3)}$

Solution : $\frac{1}{12} + \frac{1}{20} + \frac{1}{30} = \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \frac{1}{(5)(6)} = \frac{3}{(3)(6)} = \frac{1}{6}$

138. $200_7 =$ ________ $_{10}$.

Step #1 : 2(7) = 14
Step #2 : 14 + 0 (this is the middle zero) = 14
Step #3 : 14 x 7 = 98
Step #4 : 98 + 0 (this is last zero) = 98

Example A : $43_8 =$ ________ $_{10}$.

Step #1 : 4 x 8 = 32
Step #2 : 32 + 3 = 35

85. 234 base 10 equals ________ base 5.

Solution : Step #1 : 234 ÷ 5 = 46, remainder 4. Write down the 4 as first digit of answer.

Step #2 : 46 ÷ 5 = 9, remainder 1. Write down 1 as the second digit of answer.

Step #3 : 9 ÷ 5 = 1, remainder 4, Write down 4 as the third digt of answer.

Step #4 : Since 1 can't be divided by 5, the last digit of the answer is 1.

Final answer : 1414

142. A hexahedron has ________ faces.

There are 5 regular polyhedrons. They are called the Platonic Solids.

tetrahedron has 4 faces (each face is an equilateral triangle)
4 vertices and 6 edges

hexahedron has 6 faces (each face is a square)
8 vertices and 12 edges

octahedron has 8 faces (each face is an equilateral triangle)
6 vertices and 12 edges

dodecahedron has 12 faces (each face is a regular pentagon)
20 vertices and 30 edges

icosahedron has 20 faces (each face is an equilateral triangle)
12 vertices and 30 edges

143. The measure of each of the exterior angles of a regular decagon is __________ degrees.

Note : The measure of the exterior angle of an n-gon = $\frac{360°}{n}$

$$\frac{360°}{10} = 36°$$

145. The measure of an interior angle of a regular decagon is __________ degrees.

Note : An interior angle and an exterior angle are supplementary (the sum of their measures is 180°). If n is the number of sides of a regular polygon, then $\frac{360°}{n}$ is equal to the number of degrees of the exterior angle. To find the interior angle, subtract the exterior angle from 180°.

Solution : $180° - \frac{360°}{10} = 180° - 36° = 144°$

150. The 12^{th} triangular number is __________.

The nth triangular number is $\frac{n(n+1)}{2}$. The first few triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Solution : $\frac{12(12+1)}{2} = \frac{12}{2}(13) = 6(13) = 78$

151. The 9th triangular number added to the 10th triangular number equals __________.

Rule : The nth triangular number added to the (n + 1)th triangular number equals to $(n + 1)^2$.

Solution : $10^2 = 100$

153. $\frac{1}{11} + \frac{1}{121} + \frac{1}{1331} =$ __________.

Note : The denominators are powers of 11 and all the numerators are one.

Step #1 : Add sum of the two leftmost denominators to the numerator of the rightmost fraction. This result will be the numerator of the answer.

$11 + 121 + 1 = 133$

Step #2 : The denominator of the answer is the rightmost denominator.

1331

Answer : $\frac{133}{1331}$

155. $\frac{24^2 + 24}{25}$ = ________.

Solution : $\frac{24(24+1)}{25} = \frac{24(25)}{25} = 24$

156. Approximate : $59 \times 61 + 58 \times 62$ = ________.

Note : The average of the two numbers on the left and the average of the two numbers on the right are the same.

Rule : Step #1 : Find the average of the two numbers on the left.

$$\frac{59+61}{2} = \frac{120}{2} = 60$$

Step #2 : Square the result from Step #1.

$60^2 = 3600$

Step #3 : Double the result from Step #2.

$2(3600) = 7200$

Range : 6836 - 7554

157. When the sides of a square are tripled, by what factor is the area multiplied? ________.

Solution : Since you want to find out by what factor the area will be multiplied and area is square units, the factor is

$3^2 = 9$

Example A : When the radius of a sphere is tripled, by what factor is the volume multiplied? ________.

Solution : Since you want to find out by what factor the volume will be multiplied and volume is cubic units, the factor is

$3^3 = 27$

159. $\sqrt{72(50)}$ = ________.

Solution : $\sqrt{72(50)} = \sqrt{36(2)(50)} = \sqrt{36(100)} = \sqrt{36}\ \sqrt{100} = 6(10) = 60$

164. $7! \div 5!$ = ________.

Solution : $7! \div 5! = \frac{7!}{5!} = \frac{(7)(6)(5)(4)(3)...}{(5)(4)(3)...} = (7)(6) = 42$

165. The discriminant of $3x^2 - 2x + 1 = 0$ is ________.

Note : If $ax^2 + bx + c = 0$, the discriminant is $b^2 - 4ac$.

Solution : $(-2)^2 - 4(3)(1) = 4 - 12 = -8$

169. If $\frac{a}{7}$ has a remainder of 4 and $\frac{b}{7}$ has a remainder of 2, then $\frac{ab}{7}$ has a remainder of________.

Rule : If $\frac{a}{c}$ has a remainder of r and $\frac{b}{c}$ has a remainder of t, then the remainder of $\frac{ab}{c}$ is equal to the remainder of $\frac{rt}{c}$.

Solution : Find the remainder of $(4)(2) \div 7$. $8 \div 7 = 1$, remainder 1. Answer is 1.

Answer is 3.

172. How many positive integers that are less than 15 are relatively prime to 15?

Note : Relatively prime numbers : Two numbers are relatively prime if their GCD is 1.

Step #1 : Find the distinct prime factors of the number.

$15 = 3(5)$

Step #2 : Find the product of the differences of the reciprocal of the distinct prime factors and 1.

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)$$

Step #3 : Multiply the given number by the result from Step #2.

$\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)(15) = \left(\frac{8}{15}\right)(15) = 8$; The following numbers are less than 15 and relatively prime to 15 : 1, 2, 4, 7, 8, 11, 13, and 14.